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Understanding and Mathematical Fictionalism[†]

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In a recent paper in this journal, Mark Balaguer develops and defends a new version of mathematical fictionalism, what he calls 'Hermeneutic non-assertivism', and responds to some recent objections to mathematical fictionalism that were launched by John Burgess and others.
In this paper I provide some fairly compelling reasons for rejecting Hermeneutic non-assertivism — ones that highlight an important feature of what understanding mathematics involves (or, as we shall see, does not involve).

In a recent paper in this journal, Mark Balaguer [2009] develops and defends a new version of mathematical fictionalism, what he calls 'Hermeneutic non-assertivism' (henceforth, 'HNA'), and responds to some recent objections to mathematical fictionalism that were launched by John Burgess [2004] and others. If correct, HNA is supposed to possess all of the benefits of the family of fictionalist accounts of mathematical discourse without the apparent costs of endorsing accounts that fall under either of its genuses, *viz*., Revolutionary or Hermeneutic fictionalism, at least as those views are normally understood. If HNA is a viable alternative to the extant types of fictionalism, this is important news.

In his closing remarks [*ibid*., p. 161], Balaguer notes that while he does not 'know whether [HNA] is true, ... it seems to [him] that it might be true.' And he goes on to note that '[he does] not see any reason to reject it either.' As I will show, there are some fairly compelling reasons for rejecting HNA, ones that highlight an important feature of what understanding mathematics involves (or, as we shall see, does not involve).

The plan is as follows. §1 introduces philosophical fictionalism, with §1.1 discussing mathematical fictionalism, introducing Burgess's objection to that view and setting out Balaguer's response to Burgess's objection. §1.2 considers Balaguer's defense of revolutionism. After assessment, §2 introduces HNA and §2.1 sharpens the view. §3 and §3.1 raise objections to HNA, §4 considers a possible defense for HNA and §5 concludes.

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1. Mathematical Fictionalism

Before turning to mathematical fictionalism, a few words on 'fictionalism'. It is useful to distinguish *the philosophy of fiction* from *philosophical fictionalism*. The former is concerned with issues and questions that are about certain aspects of fiction, *e.g.*, about whether fictional characters exist (*e.g.*, Sherlock Holmes), about the aletheic status of claims from a work of fiction, *etc*. By contrast, rather than analyzing philosophical issues that are about certain aspects of fiction, philosophical fictionalism employs the notion of *fiction* in order to resolve certain philosophical puzzles or paradoxes that arise from some putatively problematic discourse. Our focus, for what follows, will be on one feature of philosophical fictionalism, mathematical fictionalism.

Balaguer [2009, p. 135] understands mathematical fictionalism as the view that:

(i) Mathematical sentences should be read at face value and should be read as making straightforward claims about abstract objects (*e.g.*, '4 is even' should be read as being of the form Fa and as being *about* an abstract object, the referent of '4');[1]
(ii) There are no such things as abstract objects; and, so,
(iii) Mathematical sentences are not literally true.

Although some putative fictionalists will take issue with the claim that (i)–(iii) capture *mathematical fictionalism* (*e.g.*, Stephen Yablo [2005] would take issue with (i) and (iii), since he will deny that the 'serious' content of mathematical sentences — what those sentences actually say about the world — involves reference to abstract objects), we will grant that fictionalists are committed to (i)–(iii) and will now consider two species of mathematical fictionalism, together with a challenge to the view, launched by Burgess [*ibid.*].

1.1. *Varieties of Mathematical Fictionalism*

Burgess [*ibid.*] highlights a dilemma for advocates of mathematical fictionalism: such advocates will have to endorse either Hermeneutic or Revolutionary fictionalism, but neither version of mathematical fictionalism can be endorsed; therefore, mathematical fictionalism cannot be endorsed. In response, Balaguer argues that both of these claims are false, in which

[1] Balaguer [2009, p. 132] contends that it is because mathematical sentences should be read at face value that they make straightforward claims about abstract objects. This is incorrect, however. Since the reason they are to be taken at face value in no way determines what the referents of their terms are to be, even if we grant that mathematical sentences should be read at face value, that does not provide us with a reason for taking them to make straightforward claims about abstract objects.

case Burgess's conclusion can be resisted. The first claim is addressed in §2, where HNA is introduced. The second claim is addressed below, after briefly sketching the two strands of mathematical fictionalism.

Balaguer [*ibid.*] reads 'Hermeneutic fictionalism' as the view that mathematicians intend their utterances of mathematical sentences (henceforth, 'M sentences') to be interpreted non-literally. As Burgess [*ibid.*] describes the position, mathematicians intend their talk to be taken as a form of fiction, so that, *e.g.*, a singular term like '4' is not meant to refer and a sentence like '4 is even' is not supposed to be literally true. Hence, we can take *Hermeneutic fictionalists* to hold that mathematicians intend their mathematical talk to be taken as a form of fiction.[2]

Balaguer reads 'Revolutionary fictionalism' as the view that while mathematicians do not intend their utterances of M sentences to be taken as fiction, we should treat their utterances as making assertions that are about, or that purport to be about, mathematical objects.[3] However, since there are no such objects, the assertions that they make are simply mistaken, *viz.*, they are false or, at least, are untrue. As Burgess [2008, p. 57] notes, this form of revolutionism '[denies] while doing philosophy what is asserted while doing mathematics, but not pretending that it never *was* asserted, or pretending that it was only asserted but was not really *meant* or *believed*.' (italics original) Hence, we can take *Revolutionary fictionalists* to hold that what mathematicians say when they utter their M sentences is not true.

Burgess [*ibid.*] argues against both forms of mathematical fictionalism. He rejects Hermeneutic fictionalism on grounds that the view is both wholly unmotivated and wildly implausible. He [*ibid.*] rejects Revolutionary fictionalism on grounds that 'it seems unacceptable for [Revolutionary fictionalists] to dictate to experts what they ought to say on points about which they are experts; most notably . . . , they should not dictate to mathematicians what they should say about mathematical questions.' In particular, where there is a *clash* — between what mathematicians claim and what the Revolutionary fictionalist's contend — we should side with the mathematicians and, thus, should resist the dictates of Revolutionary fictionalism.

[2] Again, insofar as the view is attributed to Yablo, it is incorrect to say that speakers intend their math-talk to be taken as a form of fiction. For Yablo [2001; 2005] is fairly explicit about the fact that mathematicians are not taking their talk as a form of fiction. Yablo's official line (*cf.* [2001]) is that while ordinary speakers do not take direct attitudes towards supposedly fiction-involving discourse, they are, in a particular sense, *simulating* belief in the surface commitments of their claims, without actually taking on any of them. That is, it is only *as if* they believe it, except, perhaps, *per accidens*, which is to say that if they believe it, the reasons are independent of their *as if* beliefs.

[3] Take M sentences to be standard mathematical sentences, *e.g.*, '4 is even', sentences of a mathematical theory, *etc*. The idea then is that their level-one claims are not true.

Balaguer agrees that, in general, this is a good methodological rule to adopt. But he argues that the present case constitutes an exception to that rule, given that there is nothing unacceptable about the ways in which fictionalists disagree with mathematicians. Put differently, according to Balaguer there is no such clash, at least no clash of the sort Burgess had in mind, *e.g.*, one that would call for a radical revision to mathematics or to the attitudes held by mathematicians towards mathematical discourse. Of course, it is possible that mathematicians and philosophers hold at least some divergent views. But any resultant disagreements would not be viewed as objectionable from the perspective of either of the two parties, for such a disagreement appears to come in at the level of metaphysics about which mathematicians have neither a stake nor any sort of expertise. As such, while Balaguer agrees with Burgess that if there were to be a genuine, or a *serious*, clash, then we should side with the mathematicians; he also maintains that no such clash is evident. From that, he concludes that Burgess's objection to Revolutionary fictionalism fails with the result being that even if fictionalists are committed to revolutionism, they are not committed to an unacceptable sort of revolutionism.

1.2. *Evaluating Balaguer's Defense*

Balaguer [*ibid.*] seems to be right that there is no serious clash. We might also add that the sorts of questions that fictionalists are answering are simply not the sorts of ones that we find mathematicians asking. Hence, it looks like Burgess's reasons for rejecting Revolutionary fictionalism can be resisted. Now, while Balaguer [2009, p. 153] grants that Hermeneutic fictionalism seems to be false, rather than going on to support Revolutionary fictionalism, he puts forward, and attempts to defend, a new type of mathematical fictionalism, HNA. In the next section, I turn to that novel brand.

2. Understanding HNA

According to Balaguer [*ibid.*, p. 157], both Revolutionary and Hermeneutic fictionalism assume that there is something that typical mathematicians *say* when they utter the M sentences that they do. But fictionalists can avoid this central feature (and, hence, can avoid a commitment to either Revolutionary or Hermeneutic fictionalism) provided they hold that when typical mathematicians utter M sentences, they do not actually assert the propositions that they appear to express. Accordingly, Balaguer contends that if fictionalists want to avoid both strands (and, in particular, avoid revolutionism), they *must* endorse HNA, *viz.*,

> *Hermeneutic non-assertivism*: When typical mathematicians utter M sentences, they should not be interpreted as saying — that is, as asserting — any proposition at all.

This brand of fictionalism might seem to be a version of what is sometimes called 'force fictionalism'. On that view, the content expressed in an (ordinary) utterance of a sentence is not asserted; instead, some other speech act is being performed. But HNA is not a form of force fictionalism, since, as the position is presented, it is not part of the view that typical mathematicians are performing some speech act *other* than asserting. Rather, the view is that while typical mathematicians might well take themselves to be asserting, they are simply mistaken about what they are actually doing. But the mistake does not concern *what* they are asserting; it regards *that* they are asserting. As Balaguer [*ibid.*, p. 158] claims, according to HNA, 'typical mathematicians do not assert what their sentences and theories actually say and, in fact are not making assertions of any kind.'

If typical mathematicians are not actually asserting then what on earth are they doing? Balaguer would likely claim that this is the wrong question to ask, for HNA does not hold that typical mathematicians are pretending to assert, since that would commit an advocate of HNA to the sort of hermeneutic view that Balaguer outright rejects. But now we should ask: why think that typical mathematicians are not asserting, when they sincerely utter the M sentences that they do? Put differently, what is the empirical support for HNA?

Balaguer [*ibid.*, p. 159] proposes an answer to this question, noting that

> [advocates of HNA] can maintain ... that the best overall account of what [mathematicians] *are* doing, given all the facts about mathematical practice and all the intentions of typical mathematicians, holds that they are not asserting what those sentences say (and, indeed, not making assertions of any kind).' (emphasis original)

I think that we can further clarify Balaguer's answer to this question and, in so doing, will further explain what lies behind HNA.

2.1. *What Typical Mathematicians Do Not Understand*

The presupposition behind the claim that mathematicians are not asserting is that there are conditions that have to be met in order for an assertoric utterance to count as an assertion and that mathematicians do not satisfy those conditions. Although Balaguer does not say what those conditions are, he does say something suggestive. He [*ibid.*] says, when discussing HNA, that mathematicians do not assert what the sentences they utter *say*. Now, as we will see, 'say' is ambiguous in an important respect. But, on one reading of it, what Balaguer's claim amounts to is that mathematicians do not assert what their sentences appear to say about the world because that natural numbers are abstract objects is not part of their *full conception of natural numbers* (henceforth, 'FCNN'). So, we might see the view as claiming that because mathematicians do not (or, at least, do not need to) believe or think about what the sentences they utter say about the world, we should not interpret their utterances as assertions.

There is another way of making this point. One might claim that because, in a certain sense, mathematicians do not understand or believe what their M sentences say, it follows that they are not asserting. Indeed, it seems that an advocate of HNA must claim that typical mathematicians neither believe nor understand what the sentences they utter say, for if they did possess such understanding and belief then it would be very difficult to maintain that they are not asserting, when they sincerely utter such sentences. So, it seems that, in order to endorse HNA, fictionalists must hold that typical mathematicians neither understand nor believe what the M sentences that they utter say about the world. We might thus read HNA as follows:

> *HNA**: When typical mathematicians utter M sentences, they should not be interpreted as either understanding or believing what those sentence say about the world and should not be interpreted as saying — that is, as asserting — any such propositions at all about the world.

Is this new version of HNA problematic? Not obviously. Indeed, read in this way, we might see an advocate of HNA as offering an understanding of mathematical practice whereby mathematicians believing what the M sentences they utter say, or understanding the metaphysical content of those sentences, is not relevant to their ability to work as mathematicians. Balaguer [*ibid.*] for example, argues that mathematicians are engaged in working out the consequences of their 'full conception' of various mathematical objects (*e.g.*, natural numbers, sets), where this conception need not include a picture of the metaphysical status of such things. Understanding the FCNN enough to be able to work out the consequences of this conception (*e.g.*, by deriving the consequences from axioms) therefore need not require one to *understand* the full metaphysical content of one's mathematical utterances taken as assertions of literal truths. And *believing* what the M sentences one utters say (again: about numbers as abstract objects) may be simply irrelevant to a practice that involves working out the mathematical consequences of one's mathematical hypotheses. So, from the fictionalist's perspective, asserting, believing, and understanding what M sentences *say* is simply inessential for mathematicians to engage in their respective practices.[4]

With HNA — now read as HNA* — thus understood, let's us turn to assess its plausibility.

3. Assessing Hermenuetic Non-Assertivism

If 'understanding what M sentences say' is to be read as understanding what M sentences say *about the world* (*viz.*, about numbers as abstract objects) then Balaguer is right that mathematicians do not need to believe or understand what their M sentences say, in order for them successfully to engage in the practice of mathematics. The problem is that this is not how that expression ought to be read.

As mentioned previously (§2.1), 'say' is ambiguous, for sometimes 'what a sentence says' is not to be read as what a sentence says about the world but is, rather, to be read as what a sentence *means*. On this latter reading, to believe or understand what a sentence says just is to understand what it means. And to assert what it says just is to express what it means. The problem is that for at least some expressions, it seems that one can understand what an expression means without knowing (or, really, having any attitude at all about) what it purports to denote. As I will show below, one can be conversant with, and can be said to understand, number-talk (and, more generally, math-talk) without taking on any attitude at all about the nature of numbers — about what numbers are, or about what, if anything, they could be.

To see this, consider a non-mathematical case, like one involving the truth predicate. One can be conversant with truth-talk and can (truly) believe that certain things are true without knowing or taking on any attitude at all about the nature of truth and, thus, without taking any attitude at all about what truth is.[5] Indeed, understanding 'true' does not require that one have any attitude at all about what *truth* is. Indeed, this is fortuitous, for the vast majority of putatively competent speakers of English have never even considered the question as to what truth is. As seems clear, they do not have to, in order to 'have the concept' of truth (and the same goes for a host of other expressions).

But if understanding and (competently) employing truth-talk does not require that one take on any attitude about the nature of truth then why not think the same about number-talk, *viz.*, that understanding and

[4] Thanks to an anonymous referee for helpful comments on this section.

[5] Of course, from the fact that one knows what a sentence (or expression) means, it does not follow that one can say what the sentence (or expression) means. If one had to, in order to be said to be competent with a term, few English speakers would be deemed semantically competent with respect to their vocabulary.

(competently) employing number-talk does not require that one take on any attitude about the nature of numbers? If we can hold that understanding and (competently) employing number-talk does not require that one take on any attitude about the nature of numbers, then we can grant Balaguer's point, that typical mathematicians neither believe nor understand what their M sentences purport to say (about the metaphysical status of things like numbers), without concluding that they do not understand their M sentences. And if we can conclude that they do understand the M sentences that they utter then it seems that we cannot conclude that they are not asserting.

Since, as I will argue in §3.1, understanding and (competently) employing number-talk does not require that one take on any attitude about the nature of numbers, my official position will be that HNA is false and that its falsity stems from an incorrect view about what understanding math-talk consists in. Hence, my suggestion will be that HNA goes wrong by insisting that mathematicians can be asserting the M sentences that they (sincerely) utter only if they understand the nature of numbers (*viz.*, to the effect that if there are numbers then they are abstract objects).

But why think that mathematicians understand their M sentences? As I will now show, we can get a sense for how mathematicians can understand the M sentences without believing (or, indeed, taking any attitude at all about) what the expressions contained therein purport to denote.

3.1. *Understanding Our M Sentences*

Balaguer [*ibid.*, pp. 142–145] provides a discussion of what we might plausibly take to involve understanding the M sentences, when he explains the FCNN, which is so-called because it provides the *full* conception of natural numbers. According to Balaguer [*ibid.*], what is built into the FCNN are all of the axioms of standard arithmetical theories, in addition to their theorems. As he [*ibid.*, p. 144] notes, it is plausible to suppose that the FCNN goes beyond axioms and theorems. So, for example, '0 is a number' is a part of the FCNN as are arithmetical truths, like '$7 + 5 = 12$'. Balaguer also claims that for one to have or possess the conception of natural numbers is for one to accept the sentences in the FCNN. But, on a straightforward reading of understanding, for one to understand the number-talk that occurs in M sentences just is for one to possess the full conception of natural numbers. And to understand such M sentences just is to understand the numerical terms that occur therein.

Now, Balaguer [*ibid.*] contends that mathematicians accept the FCNN. My suggestion is that grasping the FCNN (and, thus, accepting the sentences in the FCNN) just is an important part of what it takes for mathematicians to understand number-talk.

Balaguer grants that mathematicians have the full conception of natural numbers, but acknowledges that they should not be taken to know (or, again, have any attitude at all about) what their M sentences purport to say about the world. Thus, insofar as one who has the full conception of the natural numbers can be said to understand number-talk, it follows that mathematicians' understanding number-talk is compatible with their not taking any attitude at all about what their number terms purport to be about. And if understanding one's M sentences is sufficient for us to interpret one as asserting, when one (assertorically, sincerely) utters the M sentences that one does, then it seems to follow, *contra* HNA, that mathematicians, when they utter the M sentences, are asserting. That is, they can understand and, so, can assert their mathematical sentences without taking on any commitments at all regarding what the terms that occur therein appear to be *about*. For these reasons, HNA should be rejected.

4. Understanding and the Truth of Mathematicians

In the course of responding to a proposed objection to HNA, *viz*., that a reason for thinking that typical mathematicians are really asserting is because they take their mathematical utterances to be true, Balaguer [*ibid*., pp. 159–160] offers to fictionalists what amounts to a defense of HNA.[6] He [*ibid*.] claims that a fictionalist might grant that mathematicians think that their utterances are true in some sense, while maintaining that they do not 'have in mind' the right kind of truth for the sentences in question. In particular, they do not have in mind the kind of truth that fictionalists have in mind, which 'requires accurate descriptions of actually existing objects'. As he [*ibid*., p. 160] notes, '[i]t may be that while mathematicians think their [M sentences] are 'true' in some sense, they simply have not put any significant thought into what that amounts to.' And he [*ibid*.] goes on to conclude that if mathematicians do not think that their M sentences are true in this sense then 'it is not clear that we have any reason here to think that mathematicians are asserting what their theories say.'

I am not sure if all fictionalists have in mind the kind of truth that Balaguer envisages, for the one that he mentions seems incompatible with *deflationary* accounts of truth. But leave that aside. Balaguer is right that, in general, mathematicians have not put much thought into the property of truth (much less whether the predicate expresses one or not). But neither has nearly anyone else!

[6] To be sure, Balaguer is not explicitly advocating HNA. But he is putting it forward as a possible alternative that fictionalists could endorse; he has claimed that if fictionalists wish to avoid Revolutionary and Hermeneutic fictionalism then they *must* endorse HNA; and he is trying to provide a defense of the view, by showing that a possible objection to it fails.

So, why think that one must have any kind of truth in mind in order to be competent with truth-talk? More directly, why think that understanding 'true' requires that you know what truth is? I can see no reason for thinking that it does. It seems clear that one need not have any view at all about the nature of truth in order to be (linguistically) competent with that expression. So, being linguistically competent with the truth predicate simply does not require that one have a view about the property that the predicate appears to denote — having the concept of truth does not require that one know or, indeed, have any thoughts at all about what truth is. If it did, people would lack competence for a large part of their natural language. But, at least *prima facie*, they do not lack such competence.

What is the upshot? If understanding truth-talk does not require that one have a metaphysical view about truth (and I would agree that it does not), then we have no reason for thinking that because mathematicians do not have the 'right' kind of truth in mind, it follows that they are not asserting their M sentences. So Balaguer's response to the objection fails and we have a further reason for resisting HNA.

5. Concluding Remarks

What we have found is this. First, we have a reason — two, in fact — for rejecting HNA, thereby answering Balaguer's (implicit) question. But, second, as I have suggested, for a number of expressions, understanding them does not require that we take any attitude about what the terms that occur therein purport to denote. This has wider applications, but developing these points would take us too far afield from the topics addressed in this paper. Hence, I conclude, having provided reasons for rejecting HNA on grounds that it is based on a mistake regarding what understanding math-talk consists in.

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