Analysis Advance Access published December 8, 2011

No consistent way with paradox

Bradley Armour-Garb

In 'A consistent way with paradox', Laurence Goldstein (2009) clarifies his solution to the liar, one that he has been developing, refining and extending for the last 25 years, which he touts as *revenge immune*. In addition, he responds to one of the objections that Armour-Garb and Woodbridge (2006) (henceforth 'A-G&W') raise against certain solutions to the open pair (to be discussed below) and argues that his proffered solution to the liar family of paradoxes undermines what they call the *dialetheic conjecture* (henceforth 'DC') which predicts that consistent solutions to such paradoxes will fail for one or more of the following reasons:

(i) they will be plagued by 'revenge' problems;
(ii) they will import assumptions that are unacceptably ad hoc;
(iii) they will be incomplete, in that they look plausible only for a proper subset of the semantic paradoxes;[1] or
(iv) they will unwarrantedly restrict the expressive capacity of a language.

In this article, after critically evaluating Goldstein's response to A-G&W, I turn to his proposed solution to the liar paradox, where I show that it is difficult to see how it manages to avoid the DC.

1. *Goldstein on the Open Pair*

Here is the standard Open Pair:

(1) (2) is false,
(2) (1) is false,

Goldstein (1992) has argued that the problem that (1)/(2) presents can be resolved, given an appeal to truth-value gaps. In order to motivate gappiness, he supports the following two assumptions, which A-G&W (2006) label

(DA) If each statement in the open pair has a unique truth value, then each has the opposite truth value, and

(SA) If each statement in the open pair has a unique truth value, then each has the same value as the other.

Now, if we combine (DA) with (SA), the result will be that if (1) and (2) have unique truth values then they have both the opposite and the same truth values. Since the conjunction of the consequents in these conditionals is

1 Goldstein (2009: 388) includes in 'the liar family of paradoxes' the Barber Paradox, Curry's Paradox, the variants of the Open Pair Paradox and, of course, the Liar Paradox.

Analysis Vol 0 | Number 0 | 2011 | pp. 1–10 doi:10.1093/analys/anr122

(logically) impossible, Goldstein denies their antecedents, and concludes that (1) and (2) are both without truth values.[2]

A-G&W (2006: 403–04) argue that Goldstein's solution to the open pair lacked generality, given the presence of

(3) (4) is false,

(4) is false $\rightarrow$ (3) is false,

where '$\rightarrow$' is the material conditional. The idea was this. One might argue that (1) and (2) lack truth values, given an endorsement of (DA) and (SA). But this cannot be seen as a solution to the paradox of the open pair, generally, since, although (3)/(4) likewise exhibit support for (DA), it seems that the motivation behind the application of (SA) is lost. But without (SA), Goldstein's argument fails, which raises the question of how he is going to resolve the putative pathology – either inconsistency or indeterminacy – that the open *pairs* appear to present.[3]

1.1 Symmetry restored

Goldstein's (2009) reply to the worry that A-G&W (2006) register is to attempt to 'restore' symmetry between (3) and (4), which would reintroduce (SA) and would, thereby, support his original solution.

He argues that if we enlist classical logic and classical semantics and import a rule that governs substitution, we can substitute

(5) (3) is true $\rightarrow$ (3) is false,

for (4), in which case, given that (5) is classically equivalent to '(3) is false', we can rewrite the members of (3)/(4) as (1)/(2), thereby restoring symmetry in the pair, and resolving the problem that (3)/(4) appears to pose, courtesy of (SA) and (DA).

Now, even if we were to grant that (3)/(4) could be resolved in the way that Goldstein (1992) originally proposed to resolve (1)/(2), there are other variants of the open pair, which resist such treatment.[4] In order to see this, consider

(6) (7) is true $\vee \perp$

2 Although, Goldstein (1992) did not put it this way, as he would now describe the situation, (1) and (2) fail to make statements.

3 Goldstein (2009: 378–79) points to the arguments made by López de Sa and Zardini (2007). For some reasons for resisting their conclusions, see Armour-Garb and Woodbridge 2010.

4 The presence of the new sort of pair also undermines Priest's (2005) response to the paradox of the open pair, though, for reasons of space, I shall not establish that in this article.

(7) (6) is true $\vee \perp$,

or its asymmetrical analogue,

(8) (9) is true

(9) (8) is false $\rightarrow \perp$,

where '$\perp$' stands for absurdity, '$\vee$' represents disjunction and '$\rightarrow$' is as before.

A key difference between (6)/(7) and (8)/(9) is that they cannot be 'rewritten' as (1)/(2), even if we allow for substitution of material equivalents and work within the bounds of classical logic (and classical semantics). As such, they cannot be resolved in the way that Goldstein (1992, 2009) (or Priest (2005)) proposed to resolve (1)/(2) (and, thus, (3)/(4)).

In addition, (DA) and (SA) cannot be enlisted, in order to resolve these new pairs, since if the members of (6)/(7) or (8)/(9) are assigned divergent truth values then inconsistency results. Hence, for the two new pairs, (DA) is simply false. Moreover, the members of both of the new pairs can be assigned the same truth value, though it is indeterminate, which value they would receive. As a result, Goldstein's (2009) strategy cannot be extended to resolve the paradox of the open pair, contrary to what he has claimed.

Although Goldstein's failure to resolve the open pair paradox is problematic, he (2009: 380) does offer a more general sort of solution to the 'liar family of paradoxes'. As he (2009: 380) notes, his solution to the liar paradox can be employed as a means for resolving the paradoxicality that *all* of the members of that family appear to present, of which the open pair is an instance. As such, in order to consider the plausibility of his general solution to the semantic paradoxes, I turn to his solution to the liar paradox.

2. *Goldstein and liar sentence tokens*

Goldstein contends that sentence tokens – that is, concrete occurrences, actual inscriptions or utterances – inherit their truth values, as well as their truth conditions, from those of the statements that are made through our declarative utterances of them, where statements are taken to be the contents that get expressed, when we say or think what we do.

He (2009) argues that a liar sentence token, e.g.

(S) (S) is not true,

does not make a statement and, thus, possesses neither truth conditions nor a truth value.[5]

5 Goldstein endorses a variant of the 'no proposition' approach. For more on this approach, see Armour-Garb 2012.

Here is a version of his argument, to the effect that a liar sentence token does not make a statement, as presented in his (2009):

> Suppose that 'S' names a statement. Since, necessarily, all statements possess (consistent) truth-values, it follows that S is either true or false. Clearly, S has a truth-value that is different from that of the statement that S is not true. As such, and given a commitment to consistency, 'S' cannot name the statement that S is not true. But 'S' was chosen arbitrarily; thus, there cannot be a statement that says, of itself, that it is not true. (Goldstein 2009: 385)

Now consider a token of (S). This sentence token, if it makes any statement at all, makes the statement that this very statement is not true. But, as above, no statement can say of itself that it is not true; thus, this sentence token does not, because it cannot, make a statement.

3. *Revenge for Goldstein?*

Goldstein (2009: 386) is clear that one of the virtues of his position is that although 'This sentence token does not make a true statement' does not make a true statement we can truly express as much, by uttering a different sentence token of the same type, which makes the statement that this sentence token does not make a true statement. This is important. As is familiar from consistent solutions to the paradoxes, it would be a real deficit to Goldstein's solution if it managed to avoid paradox only by ridding the language of the expressive power that it appears to have.

Is Goldstein's solution to the liar paradox able to avoid any charge of inexpressibility? It is not clear that it is. In order to get to the problem, let 'K' name the sentence type found in the following sentence token:

(A) No sentence token of type K makes a true statement,

where '(A)' (i) names a sentence token that is of type K and (ii) 'says' that no sentence token that is of the type of which (A) is an instance makes a true statement. We can now show that (A) will not, consistently, be assigned either truth or falsity.[6]

To begin, suppose that a sentence token of type K is true (e.g. (A) or, perhaps, some other) and, for convenience, let us suppose that it is (A). Now, if a sentence token like (A) is true then the statement it makes is true and, since (A) is a sentence token of type K, it follows that a sentence token of type K makes a true statement.

6 For what follows assume that a sentence token is true (false) iff the statement that it makes is true (false). Since this assumption is granted by any propositionalist, it can surely be granted here.

Additionally, if a sentence token like (A) is true then it follows, given the T-schema, together with the fact that '(A)' names the sentence token, 'No sentence token of type K makes a true statement', that no sentence token of type K makes a true statement. Hence, if a sentence token like (A) is true, two things follow:

(i) A sentence token of type K makes a true statement; and
(ii) No sentence of type K makes a true statement.

Thus, we arrive at a contradiction. But (A) was picked at random from amongst sentence tokens of type K; therefore, no sentence token of type K will (consistently) be assigned the value 'true'.

Suppose, then, that a sentence token of type K (again, (A) or perhaps some other) is false and, for convenience, let us suppose that it is (A). If (A) is false then given what it says, together with some (innocuous) juggling (enlisting, for example, the falsity schema for statements, together with double-negation elimination), it will follow that a sentence token of type K (not necessary (A), of course) will be true. But, as we have seen, if a sentence token of type K is true then a contradiction results. Hence, without ensuing contradiction, a sentence token like (A) cannot be assigned the value 'false'. But (A) was chosen at random; therefore, no sentence token of type K will (consistently) be assigned the value 'false'. Thus, no such sentence token will be assigned the value 'true' or the value 'false'. But then since, on Goldstein's view, every statement is either true or false, it follows that no sentence token of type K can, and, thus, no sentence token of type K does, make a statement.

4. *Goldstein on revenge*

Although Goldstein will have to grant that no sentence token of type K makes a true statement, it seems that he cannot express this thought, by asserting that no sentence token of type K makes a true statement, without contradiction ensuing. Does this thus provide an irresolvable case of revenge? I am not sure that it does. But rather than attempting to establish that Goldstein will be unable to 'express the status' of sentence tokens of type K, I will point to a different problem, a different form of revenge. In order to make this case, I will first show that Goldstein is committed to the claim that no sentence token of type K makes a statement.

4.1 Goldstein and a minimal condition for expressibility

There are two ways to establish that Goldstein is committed to the claim that no sentence token of type K makes a statement. First, in a number of places, he *says* explicitly that liar sentence tokens (and kin) do not make statements. So, for example, he says, 'the sentence 'This sentence does not make a true statement' does not make a statement.' And he notes that for the sentence, 'This sentence either makes a false statement or no statement at all', we can consistently (and, thus, truly) assign to it the 'value' 'cannot be used to make a statement' (2009: 386).

Leaving aside the fact that 'value' here has merely a metaphorical use (as opposed, say, to 'semantic value' or 'truth value'), it is clear that if, as he claims, the sentence, 'This sentence either makes a false statement or no statement at all' cannot be used to make a statement then he will likewise accept that 'This sentence either makes a false statement or no statement at all' does not make a statement.

Secondly, that a sentence token of type K does not make a statement is entailed by his account. Here's how. On Goldstein's view, if a sentence token makes a statement then it is a truth bearer and if it is a truth bearer then it will have a truth value. But, as we have seen, no sentence token of type K can, consistently, be assigned truth or falsity. Hence, no sentence token of type K will have a truth value in which case, since they won't be truth bearers, on Goldstein's view, it follows that no sentence token of type K makes a statement. So, it is a consequence of Goldstein's view that no sentence token of type K makes a statement.

Given those points, it seems that Goldstein will accept and, thus, must to be able to assert

(F) No sentence token of type K makes a statement,

if his proposed solution is to be *conveyable*.[7] Let us say that this is a minimal condition for Goldstein to express his solution to the Liar family of paradoxes, a 'minimal condition for expressibility'.

As I will now show, Goldstein will be unable to satisfy this minimal condition. In more detail, what I will show is that if he is committed to a token like (F) then we can show that a token of type K will be true. But, rather than relying on intuition, plausibility, or the like, in order to support this claim, I will appeal to linguistic considerations. Since the linguistic considerations to which I will appeal are unfamiliar to some, I will explain them in some detail. Readers familiar with generalized quantifiers and the monotonic features of determiners, generally, can either skim the next section or move directly to §6.

5. Monotonicity and the semantics for a few determiners

As is familiar from work on generalized quantifiers, certain determiners possess a certain semantic property, which Peter Larson (1995) calls 'directional entailingness'. That is, certain determiners are monotone increasing (or 'upward entailing'), others are monotone decreasing (or 'downward entailing'), and still others are neither, viz., are non-monotonic. Our concern for

7 Goldstein (personal communication) has verified that, indeed, this is what he would say.

what follows will regard this one semantic property for the category of natural language determiners.

In order to keep things manageable, I restrict the discussion to 'no', where we will see an interpretation for that lexical determiner. After doing so, I will identify the semantic properties of directional entailingness and will briefly review a test, which is standardly taken by linguists to determine which semantic properties are possessed by which of the aforenoted lexical determiners. In order to illustrate the test and provide its results, I will work with a simple (and harmlessly incomplete) version of standard model-theoretic semantics.

5.1 Modelling the lexical determiners

The lexical determiners, 'every', 'some' and 'no' are seen as functions with 'determiner meaning' amounting to relations between sets or functions that apply to one set to give a function from sets to truth values or, equivalently, a set of sets.

In order to define the function for 'no' in Generalized Quantifier Theory, we begin with a basic set of individuals, A, which is the set of entities in our model, M. Associate our (common) nouns and (intransitive) verbs with subsets of A. We match determiners – in our case, 'no' – with binary relations on sets of A, in particular, associating the determiner with the following relation:

$[[\text{no}]] = \text{NO}$, where, for any sets $X, Y \subseteq A$, $\text{NO}(X)(Y) = T$ iff $X \cap Y = \varnothing$[8]

I will now proceed to our definitions for determiner meaning, which will enable us to identify certain properties that are possessed by the lexical determiner 'no':

(1) A determiner, D, is 'left monotone increasing' (otherwise called left upward entailing) iff whenever $A \subseteq C$, D(A)(B) entails D(C)(B).
(2) A determiner, D, is 'right monotone increasing' (otherwise called right upward entailing) iff whenever $B \subseteq C$, D(A)(B) entails D(A)(C).
(3) A determiner, D, is 'left monotone decreasing' (otherwise called left downward entailing) iff whenever $A \subseteq C$, D(C)(B) entails D(A)(B).
(4) A determiner, D, is 'right monotone decreasing' (otherwise called right downward entailing) iff whenever $B \subseteq C$, D(A)(C) entails D(A)(B).

What these definitions tell us is that in monotone decreasing determiners the substitution of a set with a subset yields a valid inference, while in monotone increasing determiners the substitution of a set with a superset yields a valid inference.

We can now show which semantic features are possessed by the determiner, 'no', by providing a test that shows various entailments to be valid

8 Note that '$[[\phi]]$' can be read as 'the interpretation of ϕ '.

(for ease, I will use '⇒' for entailment).[9] In so doing, we would establish certain monotonicity generalizations about this natural language determiner. Given our purposes, I will focus entirely on 'no', where I will show that it is (both left and right) monotone decreasing.

To show that 'no' is left monotone decreasing, we can employ a test for validity like the following:

(i) [[analytic philosopher]] ⊆ [[philosopher]]
(ii) No philosophers are boring ⇒ No analytic philosophers are boring.

Since the set of analytic philosophers is a subset of the set of philosophers, and since (ii) is valid, it follows that 'no' is left monotone decreasing. Semantically speaking, this means that we can substitute 'analytic philosophers' for 'philosophers' in 'No philosophers are boring' and truth will be preserved. (The relation is semantic; it is not syntactic.)

To show that 'no' is right monotone decreasing, consider:

(i) [[analytic philosopher]] ⊆ [[philosopher]], and
(ii) No dogs are philosophers ⇒ No dogs are analytic philosophers.

Since this entailment goes through, it follows that 'no' is right monotone decreasing, right downward entailing.[10]

The above thus captures the 'directional entailingness' of 'no'. With that understood, let us return to a sentence token like (F), so that we might address the question of whether Goldstein's account will satisfy the minimal condition for expressibility.

6. Expressibility and the monotonicity generalization for 'no'

In §4, we saw that on Goldstein's account it will end up that no sentence token of type K makes a statement. In §5.1, we saw that 'no' is both left and right monotone decreasing. This becomes significant, when we consider that, on Goldstein's account, the set of true statements is a subset of the set of statements. For if the set of true statements is a subset of the set of statements and if 'no' is right monotone decreasing then the following entailment is valid:

(V) No sentence token of type K makes a statement ⇒ No sentence token of type K makes a true statement.

9 Although there are myriad accounts of *entailment* that are available, for purposes of this article we shall follow those theorists who work on determiner meaning and will read 'entails' in such a way that if D(A)(B) entails D(C)(B), then truth is preserved, in the inference from D(A)(B) to D(C)(B).

10 As is easy to verify, 'no' is neither left- nor right-monotone increasing. I leave that as an exercise for the (interested) reader.

But recall the significance of '⇒'. What it tells us is that there is a truth-preserving inference from a sentence token like (F) to a sentence token like

(G) No sentence token of type K makes a true statement,

which is to say that truth will be preserved, if we substitute 'makes a true statement', for 'makes a statement', to produce a new sentence token, which will be a token of type K.

What is the upshot? We have seen that Goldstein's committed to the view that no sentence token of type K makes a statement and, thus, to the view that a sentence token like (F) will be true. We have also seen that, given the monotonic properties of our lexical determiners, the inference from a sentence token like (F) to one like (G) is valid, in which case, given Goldstein's commitment to the soundness of the relevant logical system, a token of type K will be true. But, as we have seen, given a commitment to consistency, no sentence token of type K can be true. Hence, if, following Goldstein, we retain classical logic and classical semantics then it appears that Goldstein's account will have to go. For what follows, I consider an objection that Goldstein might lodge, as against the argument that we have just worked through.

7. *Objection and response*

What might Goldstein say in response? Given his commitment to classical semantics, he cannot deny that 'no' is right monotone decreasing.[11] As such, it seems that he will have to maintain that a sentence token like (F) does not make a statement. But, as I will now show, this he cannot do.

Even leaving aside the air of ad hocery, it seems that if he were to deny that (F) made a statement, namely, by granting that (F) does not make a statement, then he should likewise conclude that a sentence token of type K does not make a statement. But if he does that, as we have seen, a contradiction seems eminent. As seems clear, he cannot contend that (F) does not make a statement while refusing to contend that a sentence token like (G) does not make a statement. Thus, his position must be that although (F) and sentence tokens of type K do not make statements, we cannot *say* that they do not make statements, in which case he is stuck with the very sort of expressibility problem – (iv), of the DC – that he took pains to avoid.[12]

11 He also cannot deny the general validity of the claim that 'no' is right monotone decreasing, even if he were to dispense with classical semantics. For some reasons, see Armour-Garb 2012.

12 Thanks to Laurence Goldstein for helpful and enjoyable discussions.

University at Albany – SUNY
Albany, NY, USA
armrgrb@albany.edu

References

Armour-Garb, B. 2012. The monotonicity of 'no' and the no proposition view. *American Philosophical Quarterly* 72: 1–14.

Armour-Garb, B. and J. Woodbridge. 2010. Truthmakers, paradox and plausibility. *Analysis* 70: 1–13.

Armour-Garb, B. and J. Woodbridge. 2006. Dialetheism, semantic pathology, and the open pair. *Australasian Journal of Philosophy* 84: 395–416.

Barwise, J. and R. Cooper. 1981. Generalized quantifiers and natural language. *Linguistics and Philosophy* 4: 159–219.

Goldstein, L. 2009. A consistent way with paradox. *Philosophical Studies* 144: 377–89.

Goldstein, L. 1992. 'This statement is not true' is not true. *Analysis* 52: 1–5.

Goldstein, L. 1985. The paradox of the liar – a case of mistaken identity. *Analysis* 45: 9–13.

Larson, R. 1995. Semantics. In *Invitation to Cognitive Science*, eds L. Gleitman and M. Liberman, vol. I: Language, 361–80. Cambridge, MA: MIT Press.

López De Sa, D. and E. Zardini. 2007. Truthmakers, knowledge and paradox. *Analysis* 67: 242–50.

Priest, G. 2005. Words without knowledge. *Philosophy and Phenomenological Research* 71: 686–94.