Liars, Truthtellers, and Naysayers: A Broader View of Semantic Pathology I

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0. *Introduction*

It is a commonplace that our linguistic capacity is reflexive; that is, we can use our language to talk about our language. One special fragment of the meta-discursive component of language is what theorists typically call *semantic discourse*. In this fragment of our talk, we make use of certain technical—specifically, semantic—notions ostensibly to express how our words and sentences "hook onto" the world in a contentful or representational manner. It is less widely appreciated that such semantic discourse is inherently *pathological*—that the ordinary, standard operation of the semantic expressions we use generate, in certain cases, particular forms of malfunctioning. The purpose of this paper is to explain this problem with our central semantic notions, a problem that, following the lead of others (Herzberger, 1970; Kripke, 1975; Grover, 1977), we will call *semantic pathology*, although our understanding of this phenomenon differs significantly from prior accounts.[1] Our first aim is to extend the recognition and discussion of this issue beyond the usual focus on the semantic paradoxes (e.g., the liar paradox). After explaining the broader problem and the challenge it poses for any attempt to account for semantic discourse, we consider some representative types of responses to (usually just part of) the phenomenon, along with the problems they face.

[1] In other work (Armour-Garb, 2005 and Armour-Garb & Woodbridge, 2006) we have distinguished the notion of 'semantic pathology' from (what we call) 'logical pathology'. In this paper, we shall focus on semantic pathology, leaving logical pathology for other discussions.

While our deeper concern is certain central semantic notions—e.g., truth, falsity, reference/denotation,[2] and predicate-satisfaction—the focus will be on their linguistic anologues: 'is true', 'is false', 'refers', 'denotes', 'satisfies', 'is true of', etc. Our reason for this is that the problem of semantic pathology manifests itself in particular linguistic cases employing these predicates, in virtue of their proper functions. The proper uses of these predicates are governed by particular principles, specifying necessary and sufficient conditions for their application. These principles make up the so-called "naïve characterizations" of the semantic predicates, as captured by the instances of the following schemata:

(T) 'p' is true if, and only if, p

(F) 'p' is false if, and only if, not-p

(R) 'n' refers to n (if it refers to anything)

(S) For all x, x satisfies 'is F' if, and only if, x is F

where 'p', 'n' and 'F' are schematic variables that get filled in with sentences, singular terms, and predicates, respectively.

These schemata are considered *standard* because of their simplicity and unrestricted scope; and they are considered *central* because any deviation from or restrictions on them will fail to capture the full expressive power of our semantic notions. This expressive power is connected to the fact that the schemata are unrestricted, meaning that there are no constraints on the subject matter of the expressions that fill in their schematic variables; they apply to our talk about anything. We will summarize this point by saying that the semantic expressions and their governing schemata are *topic neutral*. This topic neutrality is a crucial factor in these expressions fulfilling certain expressive roles that everyone should recognize as at least part of the linguistic and logical behavior of the semantic predicates.

[2] We will make no distinction between these two notions here.

The truth-predicate's role in *opaque endorsement*[3] is the paradigm for the relevant sort of expressive role: using the truth-predicate one can endorse the claim, e.g., that birds are dinosaurs without stating it explicitly (transparently), by saying "Bob's theory is true" (given the background condition of Bob's theory being that birds are dinosaurs—but regardless of whether the speaker knows this).[4] The other semantic expressions perform related roles of opaque identification and specification (for instance, via the expressions "the guy Bob referred to as 'that idiot'" and 'all people satisfying the description in the APB'). This brings out two (related) considerations that are relevant for theorizing about these semantic expressions: what is required for them to play the expressive roles they are recognized to play, and what follows from the fact that they do?

We have been suggesting that one requirement of these expressive roles is that the schemata governing the semantic expressions be unrestricted. This is a stronger claim than the common view (often the reason the above schemata are called "naïve") that the unrestricted status of the schemata is just an initial, but unnecessary assumption made prior to recognition of certain problems this generates, namely, those pertaining to semantic pathology. However, giving up this assumption "after the fact" is problematic in two senses: it raises worries about *ad hoc* theorizing, and it undercuts the expressive roles of the semantic predicates because there is no algorithm that can winnow out all and only the pathological cases and provide an effective means for restricting the schemata in the appropriate way (Kripke, 1975, pp. 691-692).

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[3] The expressions 'false' and 'not true' fulfill the role of *opaque denial*, the dual of opaque endorsement.

[4] Azzouni (2006) calls this basic role "*blind* endorsement", but we prefer 'opaque' to 'blind', since we very well might "see" what it is that we (non-transparently) endorse (or deny) with a truth (or falsity)-attribution. The contrast is with transparent endorsement, as in 'It is true that birds are dinosaurs'. Opaque endorsement also covers what is usually called the truth-predicate's *generalizing* role, since it includes endorsement of a (potentially infinite) collection of claims gathered together by a quantifier, as 'Everything Bob believes is true'. This amounts to a generalization on the embedded *sentence* positions in a claim like 'If Bob believes that birds are dinosaurs, then birds are dinosaurs'. See Quine, 1986, pp. 11-12.

The problems associated with semantic pathology gain significance when we recognize them as consequences of an assumption we must make if the semantic expressions are to fulfill their expressive functions. The relevant problems take the following general form. What we find is that, given the grammar of our language, together with the way that these expressions operate, certain sentences, or expressions, emerge for which our semantic notions malfunction, when we apply the naïve schemata to them. A general label we will use for the forms of malfunction at issue is 'resistance to semantic characterization'.[5] We will say that a case of semantic pathology arises when the standard operation of the semantic predicate it employs yields the telltale resistance to semantic characterization.

1. *Semantic Pathology I*: Liars

We begin with a summary of some of the better-known cases of semantic pathology. The most famous of these, the case that theorists have known about for over two millennia, occurs in presentations of the liar paradox, given *liar sentences*), such as

(L) Sentence (L) is false,

which self-ascribes falsity.[6] The problem here is that, at least *prima facie*, (L) is false if it is true and true if it is false. A contradiction follows directly, assuming truth and falsity mutually exclusive and exhaustive, the principle of bivalence (i.e., that every meaningful declarative sentence is either true or false), and other seemingly uncontroversial principles.[7] Thus, (L) appears to resist semantic characterization—a semantic valuation of (L)—because of an inconsistency inherent in the notions of truth and falsity.

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[5] We prefer 'semantic characterization' to, say, 'semantic *valuation*', as we do not wish to restrict the means by which we can characterize to available logical *values* (e,g., truth, falsity, both values, neither value, etc.).

[6] Alpha-numeric sentence labels are used here to denote sentence *tokens* rather than types.

[7] These principles include, centrally, unrestricted schema (T) and the operation of sentence labeling and quotation names in establishing the identity of (L) and 'Sentence (L) is false'.

One common response to the problem (L) presents is to uphold that truth and falsity are mutually exclusive, but to deny that they exhaust our available semantic characterizations. Such a theorist would deny bivalence, and would declare this and other liar sentences to be *neither* true *nor* false—to suffer a *truth-value gap.* Let us call this 'the naïve diagnosis of the simple liar'.

While this diagnosis prescribes a certain characterization—declare a problematic sentence to suffer a truth-value gap—strictly speaking, it does not (or, anyway, need not) ascribe a *truth-value* to such sentences (Kripke, 1975, pp. 700-701 fn. 18; Soames, 1998, Chapter 6, *passim*). It does, however, *semantically* characterize such a sentence—there is a fact of the matter about its semantic status: it is gappy (we might say *determinately* gappy), or neither true nor false.[8] *Prima facie*, then, the postulation of truth-value gaps resolves semantic pathology by increasing the range of semantic characterization to cover sentences it previously did not.[9]

The problem with a simple (or naïve) appeal to truth-value gaps is that there is a well-known *revenge problem* thwarting it (we shall return to the notion of revenge, in §4). If one were to deny the principle of bivalence and classify (L) as neither true nor false, she would then be faced with a *strengthened liar*,

(L+) Sentence (L+) is not true.

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[8] While it is standard to equate gappiness with indeterminacy (cf. Kleene, 1952), we do not make this identification here.

[9] This approach is typically based on the elegant (but, ultimately, unsuccessful) denial of bivalence developed in Kripke, 1975. What Kripke outlined there was a means for interpreting a language that contained its own truth-predicate. Under Kripke's 'outline', (L) was judged to be neither true nor false and, thus, as *indeterminate* or *gappy*. Kripke later notes the limitations of his model. In particular, he notes that what his theory does not yield is an interpretation for a language that contains its own *untruth*-predicate. Using the vocabulary that we favor, we can see Kripke as granting that his theory of truth did not allow for a general treatment of truth's pathology, since it could not provide a semantic characterization for a liar sentence like (L+) below, in the very language in which (L+) is formulated. We return to this issue in §4.

For an argument to the effect that we should give up any demand that (L) and (L+) be determinately gappy, see Soames, 1998, pp. 188-95.

If giving up bivalence really solves the problem that (L) presents, then the same should be true of (L+). But if we assume that (L+) is neither true nor false, then it follows (or, anyway, appears to follow) that (L+) is not true. And since that is precisely what (L+) appears to say, given the Law of Excluded Middle,[10] we can (and appear to be forced to) conclude that (L+) is both true and not true, which is a standard contradiction.[11]

Lest one imagine that the problem just rehearsed is a minor quirk arising in only a small number of specially constructed self-referential cases, it is important to note the self-referentiality of (L) and (L+) is not inherent in the sentences themselves but is an extrinsic matter, deriving from the contingent fact that they are labeled '(L)' and '(L+)'. The relevance of this point comes out more strongly when we consider cases categorized explicitly as contingent liar paradoxes, e.g., writing 'The only sentence written on the blackboard in Room 2240 of Angel Hall is not true' on an otherwise clean blackboard. If that blackboard happens to be in 2240 Angel Hall, then this sentence appears to manifest semantic pathology; if it is written on a board in some other room, then it might be true, false, or perhaps gappy (or even inconsistent: see (1) and (2) below), depending on what (if anything) is written on the board in 2240 Angel Hall.[12] The point is that it is far easier for instances of truth-talk to turn out to be pathological than one might expect, and their doing so can catch us by surprise and require empirical investigation of the particular contingent circumstances to discover.[13]

Liar sentences, such as (L) and (L+), are the most familiar loci of semantic pathology. The inconsistency they reveal in naïve truth theory is widely appreciated, as is its relentlessness.

[10] The Law of Excluded Middle is a logical principle to the effect that it is a logical truth that either a sentence, or its negation, holds: for any sentence, *p*, either *p* or not-*p*.
[11] (L+) provides a useful example of a revenge problem, as it reveals a tension between semantic universality and (what we will call) *expressive incompleteness*. In the case of (L+), the latter emerges when we realize that we cannot *say* what we clearly accept—that, whatever else we wish to say about (L+), we certainly think it is not true. For more on expressive incompleteness, see Armour-Garb, 2005.
[12] For a discussion of the contingency, see Armour-Garb, 2001.
[13] Kripke, 1975, pp. 691-692 refers to this fact as the "riskiness" of truth-talk.

As the move from (L) to (L+) thwarts an attempt to eliminate the inconsistency via an appeal to gaps, similar sorts of revenge problems stymie further attempts to provide a consistent resolution of this inconsistency.[14]

Consider, for example, a *Curry sentence* (so named because they induce the paradox presented by Haskell Curry—Curry's Paradox), for example,

(C) If sentence (C) is true, then 1 = 0.

Attempts to ascribe (C) a truth-value yield a dilemma: we face either liar-like inconsistency or *trivialism*—the thesis that every claim is true. If we assign falsity to (C), then its consequent must be false and its antecedent, 'sentence (C) is true', must be true, thereby making (C) true after all.[15] If we assign truth to (C), then inconsistency follows unless we maintain that its consequent is true. If (C)'s consequent is false, then (C) is true if, and only if, its antecedent is false as well, meaning that (C) is true if, and only if, it is false that (C) is true, i.e., if, and only if, (C) is not true. We can avoid this inconsistency only if we say that (C) is true with a true antecedent. But then its consequent must also be true, in which case, given that the consequent is '1 = 0', it follows that '1 = 0' is true. Since the choice of our consequent was arbitrary—since, that is, we could fill in *any* sentence at all (even an intuitively absurd one)—it seems to follow that every sentence is true. Given the unacceptability of trivialism (which makes any given claim and its negation both true), our Curry sentence (or, really, any other rendering of Curry's paradox) appears to resist semantic characterization, which is indicative of semantic pathology.

[14] See McGee, 1991, p. 5 on the Strengthened Liar Response. As a quick "first-pass" sample, note the following cases. Sentence (L_1) = 'Sentence (L_1) is not stably true' challenges rule-of-revision solutions. Sentence (L_2) = 'Sentence (L_2) is not definitely true' confronts indeterminacy solutions. Sentence (L_3) = 'Sentence (L_3) is not true in any context (or at any level of the hierarchy)' confronts contextual/indexical/hierarchical solutions.
[15] Here and throughout this essay, please assume that the conditional is the material conditional.

The phenomenon under consideration, what we might call *inconsistent semantic pathology*, also arises in cases beyond individual liar (or Curry) sentences, in cases that do not involve direct self-reference. For example, there are *liar loops*, such as the pair

(1) Sentence (2) is true

(2) Sentence (1) is false,

where any self-reference involved is indirect as well as contingent. Inconsistency arises in (1) and (2) much like it does in (L), just with the referential path traversing a wider (and potentially arbitrarily wide) circle (Kripke, 1975, pp. 691-693; Grover, 1977, p. 597). Still further, there are *liar series* in which self-reference appears to play no part, such as the infinite sequence generating Yablo's paradox (Yablo, 1993a):[16]

(S_1) For all $k > 1$, sentence (S_k) is false

(S_2) For all $k > 2$, sentence (S_k) is false

$\vdots$

(S_n) For all $k > n$, sentence (S_k) is false

$\vdots$

Inconsistency arises here as follows. If a sentence in the series, say (S_n), is true, then all sentences (S_k) where $k > n$ are false, including both (S_{n+1}) and all (S_k) where $k > n+1$. However, since the latter is what (S_{n+1}) says, this makes (S_{n+1}) true as well as false. If (S_n) is false instead, then it is false that all sentences (S_k) where $k > n$ are false, i.e., at least one is true. But we have already seen that if any sentence in the series is true, then the sentence after it is both true and false. So, a contradiction arises from an ascription of either truth-value to any sentence in the series, revealing a resistance to semantic characterization in every sentence in the series.

[16] On the issue of self-reference in Yablo's paradox, see Priest, 1997 and Beall, 2001b.

In the cases canvassed thus far, all of the claims involved have been instances of semantic discourse. A further, though less familiar type of inconsistent case combines semantic and non-semantic claims. Consider the following liar set, *Set (A)*.

(3) The moon is made of cheese.

(4) The Earth is smaller than the sun.

(5) Set (A) contains an odd number of true claims.

(3) and (4) are straightforwardly empirical claims involving no semantic notions. Still, contradiction arises when (5) attempts to distribute truth-values across all of the claims in (A), since, given the falsity of (3), the truth of (4), and what (5) says about them and itself, there is no consistent way for (5) to assign itself a truth-value: it is false if it is true and true if it is false.

The foregoing examples involve the notions of truth or falsity. However, the inconsistency arising from semantic pathology also appears to infect other semantic notions, such as reference, predicate-satisfaction, and even validity. In the case of reference, we can generate expressions that yield Berry's paradox, for instance, 'the least number not denotable in less than 18 syllables'. If we plug this definite description into schema (R) we get

(B) The expression 'the least number not denotable in less than 18 syllables' refers to the least number not denotable in less than 18 syllables.

The problem, however, is that in (B) an expression containing only 17 syllables manages to refer to a number that it takes a minimum of 18 syllables to denote, yielding a contradiction (Chihara, 1979, p. 599).

In the case of predicate-satisfaction we have Grelling's paradox involving the expression 'heterological'. A predicate is heterological if, and only if, it does not satisfy itself. So, for instance, the predicate ‘long’ is heterological, since it is not long. Substituting the more perspicuous ‘does not satisfy itself’[17] into schema (S) yields

> (H) The predicate ‘does not satisfy itself’ satisfies ‘does not satisfy itself’ (i.e., itself) iff the predicate ‘does not satisfy itself’ does not satisfy itself.

So, this predicate satisfies itself if, and only if, it does not satisfy itself, yet another contradiction (Ibid., pp. 597-598).

Less familiar is that liar-like inconsistency also seems to be latent in the notion of validity, as certain inferences suggest (Read, 1979 and 2001). Consider the following argument:

$$\text{(I)} \qquad \frac{1 = 1 \qquad .}{\therefore\ \text{Argument (I) is invalid}}$$

As with Berry’s and Grelling’s paradoxes, this argument seems to reveal an inconsistency in a notion other than truth or falsity. *Ex hypothesi*, the argument is either valid or invalid. Suppose that it is valid; given that the premise is true, it follows that the conclusion must be true. But if the conclusion is true then, given what it says, it follows that the argument is invalid. Thus, if the argument is valid, it is invalid. Trivially, the argument is also invalid if it is invalid, essentially proving that (I) is invalid. However, because this is what the conclusion of (I) says, and the proof relies on the truth of the argument’s premise, this shows that the conclusion follows from the premise, making (I) valid after all. Thus, (I) is valid if, and only if, it is invalid. Basic logical transformations then give us the contradiction that (I) is valid and invalid.

We have rehearsed the many ways that inconsistent semantic pathology arises for two reasons. The first pertains to extending Saul Kripke’s point about the riskiness of the notion of truth (Kripke, 1975, pp. 691-692): not only is contradiction a potential liability with many or even most of our truth-attributions, it is also a liability in our talk of reference, satisfaction, and

[17] Another version involves the predicate ‘is not true of itself’.

validity. This strengthens the point that pathological inconsistency is not limited to a small set of peculiar and easily identifiable claims about truth or falsity that we can isolate and ignore. The full scope of this inconsistency is often lost in the extensive focus on the liar paradox.

The second reason for the rehearsal pertains to demonstrating the full extent of semantic pathology, since each of the cases displaying pathological inconsistency has an analog that resists semantic characterization in a different way. The points we want to emphasize in the next two sections are (i) that semantic pathology bifurcates and so involves more than just semantic paradox, and (ii) that even the full extent of inconsistency is but one half of the problem, as this second form of resistance to semantic characterization is just as pervasive and recalcitrant as the more widely recognized one we reviewed in this section.

2. *Semantic Pathology II*: Truthtellers

While theorists continue to discuss many of the inconsistent cases canvassed in §1, what is less widely broadcast is that, in addition to their capacity to generate paradoxical cases, our central semantic notions are all pathological in another way as well. They all generate cases that manifest another sort of resistance to semantic characterization, one displayed most basically in *truthteller sentences*, such as,

(K) Sentence (K) is true.

While liar sentences like (L) appear to exhibit inconsistency, the problem with (K) is different. Here either truth-value ascription would be perfectly consistent—the assumption of (K)'s truth is sufficient for its truth, and the assumption of (K)'s falsity is sufficient for its falsity—but *prima facie* there seems to be nothing that favors (or could favor) (K)'s having one truth-value over the other. So, (K) exhibits a form of *indeterminacy* with respect to truth-value.

The problems that (L) and (K) manifest are related, since both display a resistance to semantic characterization arising from the standard operation of the semantic predicates they employ. Thus, semantic pathology appears to be a single phenomenon that gives rise to two different *symptoms*: inconsistency and indeterminacy.

The truthteller has received some attention in the literature (Herzberger, 1970, pp. 149-150; Kripke, 1975, p. 693; Grover, 1977, p. 597; Yablo, 1985, p. 300 and 1993a, pp. 386-7; Beall, 2001a, p. 126; Armour-Garb, 2001, p. 280), but what is not widely appreciated is that the indeterminacy infecting (K) extends in all of the ways that its sibling symptom, inconsistency, does. For example, like the liar's inconsistency, the truthteller's indeterminacy resists resolution via a simple appeal to truth-value gaps by recurring in what we call the *strengthened truthteller*,

(K+) Sentence (K+) is not false.

If we say that (K+) is true, then it follows that it is not false and hence, assuming bivalence, true; if we say that (K+) is false, then, given the naïve schema for falsity, together with double-negation elimination (which we assume), it follows that (K+) is false. There is still a resistance to semantic characterization, since both truth-values apply equally well, and nothing favors one over the other.

We cannot overcome this resistance simply by saying that (K+) is gappy. To begin, it is important to recognize how the indeterminacy of (K) and (K+) differs from gappiness. As we mentioned above, while saying that a sentence is gappy is not to ascribe it a truth-value, it is to characterize that sentence semantically, to ascribe it a determinate semantic status. When a sentence is gappy it seems that it is determinately *neither* true *nor* false. (K) and (K+) appear to resist semantic characterization in virtue of being *indeterminate*, by allowing *either* truth-value, while favoring neither of them.

In response, one might try to deal with (K) and (K+) by claiming that this 'either' transforms into a 'neither', meaning that, while these two sentences are admittedly odd (in fact, paradoxical in the ancient sense), they are not pathological; to the contrary, they are simply to be ascribed a gap and, as a result, end up semantically characterized. The problem with this approach to (K) and (K+) is that the latter brings out a revenge problem similar to the one that arose for the liar. Suppose that (K+) is gappy. In that case, as above, it thus follows that (K+) not true and that (K+) not false. So, in particular, (K+) is not false. But, given (T), we then get the following: 'Sentence (K+) is not false' is true. Since sentence (K+) = 'Sentence (K+) is not false', substitution of identicals yields that (K+) is true. So, if (K+) is neither true nor false, then (K+) is true. But if (K+) is neither true nor false, then it follows that (K+) is not true. Hence, if (K+) is said to be gappy, then (K+) is both true and not true, which is a standard contradiction.

Thus, an appeal to truth-value gaps does not eliminate semantic pathology from (K+). Rather, it just shifts the case at hand over to the first form of resistance to semantic characterization, inconsistency. Moreover, we might now say that (K+) also displays a higher-order indeterminacy, since it is indeterminate whether we should view (K+) as (first-order) indeterminate or inconsistent.[18]

As the strengthened liar, (L+), is mirrored by the strengthened truthteller, (K+), Curry's paradox also has an indeterminate analog, presented by sentences like

(C') If sentence (C') is false, then 1 = 0.

If (C') is false, then, since this is what its antecedent says, its antecedent is true. So, to be false, its consequent must be false, which is what we want here to avoid trivialism. If (C') is true, then its antecedent is false, given what the antecedent says. This is sufficient to make (C') true, so we

[18] We shall have more to say about this below.

can again maintain that the consequent is false, as desired. Either truth-value ascription works fine, and there is nothing favoring one over the other.

Indeterminate semantic pathology also arises in *truthteller pairs* and *truthteller series* (Kripke, 1975, p. 693; Grover, 1977, p. 597), such as the pair,

(1') Sentence (2') is true

(2') Sentence (1') is true,

and the sequence,

(S'_1) Sentence (S'_2) is true

(S'_2) Sentence (S'_3) is true

⋮

(S'_n) Sentence (S'_{n+1}) is true

⋮

Consistency holds so long as the claims in these latter cases all get the same truth-value. But since either truth-value is perfectly consistent and equally (un)motivated, as in the basic case of (K), there is no way to answer the question of which one they get.

The type of partly non-semantic case that Set (A) exemplifies above also has an indeterminate analog. Replacing the last claim of Set (A) in a particular way produces *Set (A')*, in which the last claim can be either true or false, but there is no reason to assign it one truth-value rather than the other. Set (A') contains the following sentences.

(3') The moon is made of cheese.

(4') The Earth is smaller than the sun.

(5') Set (A') contains an even number of true claims.

The falsity of (3'), the truth of (4'), and the way that (5') attempts to distribute truth-values across them and itself makes it the case that (5') could be either true or false, again with nothing favoring one truth-value over the other.

Pathological indeterminacy also extends beyond the notions of truth and falsity—to the notions of reference, predicate-satisfaction, and validity—just as inconsistency does. For the case of reference, there are expressions like ‘the greatest number denotable in less than 18 syllables’. Given that it employs 17 syllables, this expression could denote any number whatsoever (Gupta and Belnap, 1993, p. 264, fn. 23). For predicate-satisfaction, there is the expression ‘autological’ (von Wright, 1960), which applies to predicates that are true of, or satisfy, themselves (e.g., the predicate ‘short’ is autological, since it is a short expression). From the assumption that ‘autological’ is autological it follows that it satisfies itself (and so is autological), but from the assumption that ‘autological’ is not autological, it follows that it does not satisfy itself (meaning it is not autological). Either status is consistent, and nothing favors one over the other.

Indeterminacy further appears to infect the notion of validity in a way similar to how inconsistency does.[19] We can see this in the following argument.

(II) $\underline{\qquad 1 = 1 \qquad .}$
∴ Argument (II) is valid

This argument is either valid or invalid. Consider what follows from the assumption that (II) is valid. In a valid argument, the conclusion follows from the premises, so, since the premise of (II) is true, its conclusion must be true if (II) is valid. Given what the conclusion says, it follows that it is true that (II) is valid, if (II) is valid. That is, (II)’s validity follows from the assumption of its validity. On the other hand, if we assume (II) is invalid, then its conclusion is false. Again, since its premise is true, this would make (II) invalid. Thus, (II)’s invalidity follows from the assumption of its invalidity. The problem is that there seems to be nothing favoring one

[19] Our discussion of this and the other validity cases here follows Woodbridge & Armour-Garb, 2008.

assumption over the other, so we have a case where the notion of validity manifests indeterminacy.

All of the central semantic notions exhibit pathological indeterminacy. The problem this presents is that it seems that our semantic notions should cover the problematic cases, since the schemata governing them are supposed to be unrestricted. This is especially so for the semantic notions beyond truth, where the notion of a gap seems to break down. However, it appears that the only way these cases could get some semantic assignment is if we just pick one arbitrarily to eliminate the indeterminacy. Such a resolution is *prima facie* highly unsatisfying because it is blatantly *ad hoc*. This is unacceptable, at least in the context of any realist view of truth and the other semantic notions (including the sort of epistemic account of truth often called 'anti-realist'), since it is implausible to hold that an *ad hoc* account of this sort characterizes a real property.

The way that pathological indeterminacy parallels pathological inconsistency, and the fact that both forms of resistance to semantic characterization result from the ordinary operation of the semantic predicates, suggest quite strongly what we claimed above: inconsistency and indeterminacy are two symptoms of a single underlying phenomenon. The following table brings together all of the cases we have considered and highlights how the two symptoms mirror one another.

Inconsistent Cases (Liars)	**Indeterminate Cases (Truthtellers)**
• (L) (L) is false	• (K) (K) is true
• (L+) (L+) is not true	• (K+) (K+) is not false
• (1) (2) is not true (2) (1) is true	• (1') (2') is true (2') (1') is true
• (C) If (C) is true, then 1 = 0	• (C') If (C') is false, then 1 = 0
• (S_n) For all k>n, (S_k) is not true	• (S_n) (S_{n+1}) is true
• <u>Set (A):</u> (3) The Earth is round. (4) The Moon is cheese. (5) Set (A) contains an odd	• <u>Set (A'):</u> (3') The Earth is round. (4') The Moon is cheese. (5') Set (A') contains an even

Inconsistent Cases (Non-Alethic)

- ‘The least number not denotable in less than 18 syllables’ (Berry’s paradox)
- ‘…is heterological’ or ‘…does not satisfy itself’ (Grelling’s paradox)
- (I) 1 = 1 .
  $\therefore$ Argument (I) is invalid

Indeterminate Cases (Non-Alethic)

- ‘The greatest number denotable in less than 18 syllables’
- ‘…is autological’ or ‘…does satisfy itself’
- (II) 1 = 1 .
  $\therefore$ Argument (II) is valid

3. *Semantic Pathology III*: Symmetrical Naysayers

The cases of semantic pathology we have presented thus far each manifest a single symptom of this phenomenon. While the mutual mirroring of the two forms of resistance to semantic characterization suggests that they are linked, there is a third class of examples that more clearly establishes this connection via the *dual-symptom* nature of its members. These cases manifest one or the other of the two symptoms of semantic pathology, depending on what semantic ascriptions we make. The paradigm case for this class is what we have called the *open pair*:[20]

(6) Sentence (7) is false

[20] The original source for this case is Jean Buridan’s Eighth Sophism from Chapter 8 of *Sophismata*. See Hughes, 1982, p. 73. A case something like the open pair is presented in Kripke, 1975, pp. 696-697, and then cited in Grover, 1977, p. 600, but for them the issue is *levels* of truth and *riskiness*. The indeterminacy of the open pair is briefly acknowledged in Yablo, 2003, p. 319, fn. 10, where it is called “under-determination”. Detailed consideration of the open pair’s pathological nature is offered in Goldstein, 1992; Sorensen, 2001, Chapter 11 and 2003; Priest, 2005; Woodbride & Armour-Garb, 2005; Armour-Garb & Woodbridge, 2006.

(7) Sentence (6) is false.

Equally (or potentially more) problematic is the *strengthened open pair*,

(8) Sentence (9) is not true

(9) Sentence (8) is not true.

Ascribing the members of these pairs matching truth-values (either T-T or F-F) yields inconsistency. This is obvious in the case of (6)/(7); it holds for (8)/(9) given the usual connections between falsity and negation. Consistency holds so long as we ascribe the sentences of each pair divergent truth-values, that is, if we say one sentence in the pair is true and the other is false. However, because of the symmetry of the pairs, we can consistently make either divergent truth-value ascription. Since there is no obvious way to motivate one assignment over the other, indeterminacy follows. The symmetry not only generates the indeterminacy in these cases, it is also what makes the indeterminacy intractable, as it thwarts the application here of strategies sometimes employed (ineffectively in our view) in arguing for the determinate falsity of (K) (Priest, 1987, p. 84; Yablo, 1993b, p. 387).

On the model of labeling the first family of pathological cases *liars* and the second sort *truthtellers* we call this third, dual-symptom family of cases *naysayers*, since each pair presenting such a case consists of members that in effect say "nay" of one another. That is, each denies the application of the semantic concept in question (truth, reference, predicate-satisfaction, validity) to cases determined by (or simply identical with) the other.[21]

Because of the dual-symptom nature of naysayers, one can avoid inconsistency only by embracing indeterminacy. But a semantic theorist cannot sit complacently with this

[21] Although similar to Sorensen's (2001) 'no-no paradox' label for cases like (6)/(7), we prefer calling the general class of such cases *naysayers* and the paradigm case *the open pair*. This is in part because, as we show presently, the general class extends well beyond (6)/(7) and in part because even the paradigm case is not a paradox, since it is open to consistent truth-value assignments (in fact, more than one).

indeterminacy. To begin with, since we can consistently ascribe truth-values to each of (6)-(9), it seems that the theorist must make some assignment to these sentences to avoid leaving his semantic theory incomplete. Again, it looks as if the only way to make an account of truth cover all the sentences it seems like it should cover is to ascribe truth-values to these sentences arbitrarily (though consistently), and the problem with this sort of resolution of indeterminacy is that it is *ad hoc*.

The situation worsens for the semantic theorist, as there is also a naysayer case stemming from Curry's paradox, linking its inconsistency and the indeterminacy of its analog. Consider what we call the *Curry open pair*:

(C1) If sentence (C2) is true, then 1 = 0
(C2) If sentence (C1) is true, then 1 = 0.

As with Curry's paradox, trivialism is the real threat, if we ascribe (C1) and (C2) matching semantic values. If both sentences are true, then their antecedents are true, which means their consequents are true as well—whatever they may be. If both sentences are false, then their antecedents are false, making both sentences true as well. Once again, their consequents follow. While it is possible to avoid trivialism by ascribing one of these sentences truth and the other falsity, the two equally viable ways of doing this yield the usual indeterminacy. Thus, on pain of accepting trivialism, these sentences appear to resist semantic characterization, no matter how we try to ascribe semantic values to them, making it impossible to avoid semantic pathology.

Still further, the scope of the naysayer family includes cases for other semantic notions beyond truth and falsity. We can see it including predicate-satisfaction by considering the following pair of predicates:

(P1) …does not satisfy the predicate labeled '(P2)'
(P2) …does not satisfy the predicate labeled '(P1)'.

Given the unrestricted schema for the notion of predicate satisfaction, e.g.,

(S) For all x, x satisfies 'is F' if, and only if, x is F

(P1) and (P2) appear to yield either inconsistency or indeterminacy. For any object, say it satisfies both or neither of (P1) and (P2), and inconsistency follows; on pain of inconsistency, then, each object satisfies either (P1) or (P2), but not both. However, there is nothing that motivates assigning a given object to the extension of one predicate rather than the other, yielding indeterminacy. Avoiding inconsistency involves embracing indeterminacy.

The same line of reasoning applies to the naysayer version of Berry's paradox, involving the notion of reference:

(N1) The thing(s) not referred to by the expression labeled '(N2)'
(N2) The thing(s) not referred to by the expression labeled '(N1)'.

Given the unrestricted reference schema, e.g.,

(R) 'n' refers to n (if it refers to anything)

we can avoid inconsistency only if we treat each object as the referent of either (N1) or (N2), but not both. Of course, it would be indeterminate in each case which expression denotes the object.

The link between the two symptoms of semantic pathology that arguments (I) and (II) exhibit separately in the notion of validity, emerges in the following pair of arguments, which we will call the *validity open pair*:[22]

$$\text{(III)} \quad \frac{1 = 1}{\therefore \text{ (IV) is invalid}}$$

$$\text{(IV)} \quad \frac{1 = 1}{\therefore \text{ (III) is invalid.}}$$

Inconsistency follows, if we ascribe (III) and (IV) the same validity-status, viz., as both valid or both invalid. In order to see this, suppose that both arguments are valid. Then, given that 1 = 1,

[22] This case, and the arguments concerning it, are considered in Woodbridge & Armour-Garb, 2008.

both conclusions must be true, in which case both arguments are also invalid, yielding inconsistency. Thus, if the arguments are both valid, they are both invalid. Since, trivially, it is also the case that both arguments are invalid if both are invalid, it follows that if these arguments have the same validity-status: they are both invalid. However, this just is the result that (III) is invalid and (IV) is invalid. Recalling that we relied on the truth of the arguments' premises, in order to derive this, it follows that, assuming (III) and (IV) have the same validity-statuses, we can derive the conclusion of each argument, given its premise. But this means that both arguments are valid. Thus, claiming (III) and (IV) have the same validity-status yields the inconsistency that they both are both valid and invalid.

We can avoid inconsistency, if we ascribe (III) and (IV) divergent validity-statuses. For example, if we ascribe validity to (III), then since the premise is (necessarily) true, the conclusion must likewise be true. And the conclusion is true, so long as (IV) is invalid. All it would take for (IV) to be invalid, given the truth of its premise, is for it to have a false conclusion. And the conclusion is false so long as it is not the case that (III) is invalid—i.e., if (III) is valid, which is what we assumed. It is therefore perfectly consistent to maintain that (III) is valid and that (IV) is invalid. *Mutatis mutandis* for the ascription of validity to (IV), which comports with the invalidity of (III). Accordingly, while we *can* consistently make either divergent ascription, there is no obvious reason to pick one over the other, yielding the indeterminacy of validity.[23]

These are the basic naysayer cases for the central, technical semantic notions. Something noteworthy about all of the cases considered thus far is that within each pair the elements are symmetrical, that is, each 'says the same thing' of the other. This observation is the basis of

[23] Similarly, we can assume that (III) is invalid, thereby yielding the validity of (IV), and *mutatis mutandis* in the other direction.

what we call the *symmetry response* to the open pairs. It proceeds along the following lines. Consider the paradigm case of (6) and (7). Given their symmetry, any reason for ascribing a particular truth-value to one of these sentences is equally reason for ascribing that same truth-value to the other. The open pair, on this view, seems to amount to a more complex form of the liar paradox, manifesting only inconsistency, and is to be dealt with the same way that other liars are, whatever way the theorist in questions thinks that is (Goldstein, 1992; Priest, 2005).

4. *Semantic Pathology IV:* Asymmetrical Naysayers

The problem with this approach, however, is that all the open pair cases have asymmetric variants where the elements do not "say the same thing" of each other.[24] Consider the *asymmetric open pair*:

(10) Sentence (11) is false
(11) If Sentence (11) is false, then sentence (10) is false.

This pair has the same semantic features as (6)/(7). If (10) and (11) were both true, then since that would make the sentence '(11) is false' true, it would follow that (11) has a true antecedent. To be true, then, (11) would also need a true consequent, making the sentence '(10) is false' true as well. But since both '(11) is false' and '(10) is false' are true if both (10) and (11) are true, it follows that (10) and (11) would both be false if they were both true. If (10) and (11) were both false, then (10) would also be true, given what it says about (11), and (11) would be true as well, since its antecedent, (10), is *ex hypothesi* false.

We can avoid inconsistency only by embracing indeterminacy. If we say (10) is true, it follows that (11) is false, which it will be so long as it is false that (10) is false. This matches our

[24] These asymmetric cases are discussed in Woodbridge & Armour-Garb, 2005 and Armour-Garb & Woodbridge, 2006.

assumption that (10) is true. If we say that (10) is false, then '(11) is false' is false, meaning (11) has a false antecedent. So (11) is true (and so not false). Either divergent assignment works equally well, and nothing favors one over the other, yielding indeterminacy. Unlike in the earlier open pairs, however, (10) and (11) are not symmetric, so an appeal to symmetry does nothing to eliminate this indeterminacy.

The attempt to use symmetry as a reason for treating the curried open pair as just a more complicated version of Curry's paradox runs up against the *asymmetric curried open pair*:

(C3) If sentence (C4) is true, then ⊥.[25]

(C4) If sentence (C4) is true only if ⊥, then sentence (C3) is true only if ⊥.

If both of these sentences are true, then both of their antecedents are true (it being true, *ex hypothesi*, that (C4) is true, and (C3) being logically equivalent to '(C4) is true only if ⊥'). But then both of their consequents would be true, leading, in both cases, to the truth of '⊥', i.e., to trivialism. If both sentences are false, then both have false antecedents ((C4) because (C3) is its antecedent; (C3) because its antecedent says that (C4) is true when, *ex hypothesi*, it is false) and so both sentences are true. Any matching truth-value assignment here thus yields trivialism.

Divergent truth-value assignments to (C3) and (C4), on the other hand, are perfectly consistent. If (C3) is false, then it has a true antecedent and a false consequent. The latter is what we want, and the former holds good, since (C3) is the antecedent of (C4), meaning that the falsity of the former yields the truth of the latter. But (C3) could equally well be true with a false consequent, provided its antecedent is false as well. It will be, so long as (C4) is not true; and, given that it has a true antecedent (i.e., (C3)), that will follow so long as its consequent is false. Since the antecedent of its consequent is true (*ex hypothesi*), this means that (C4) as a whole is false i.e., not true) so long as '⊥' is false, which it is. So (C3) can be true and (C4) false. Either

[25] '⊥' marks absurdity, or can be glossed as a statement of trivialism, i.e., the claim 'Everything is true'.

divergent truth-value assignment works equally well, and there is nothing favoring one over the other, yielding indeterminacy—an indeterminacy untouched here by any symmetry considerations.

There are also symmetry-breaking naysayer cases for predicate-satisfaction,

> (P3) …does not satisfy the predicate labeled '(P4)'
>
> (P4) …satisfies both the predicate labeled '(P3)' and the predicate labeled '(P4)', or does not satisfy the predicate labeled '(P3)',

and, for reference,

> (N3) The thing(s) not referred to by the term labeled '(N4)'
>
> (N4) The thing(s) either referred to by both the term labeled '(N3)' and the term labeled '(N4)' or not referred to by the term labeled '(N3)'.

As with (10)/(11) and (C3)/(C4), the members of these pairs do not 'say the same thing' of one another, so there is no reason to reject divergent semantic assignments. On pain of inconsistency, each object is in the extension of one, but not both of these predicates, and each object is a referent of one but not both of these descriptions. Focusing on the predicate case, if an object satisfies both (P3) and (P4), then it also does not satisfy (P4); if an object satisfies neither predicate, then it also does satisfy (P3) in virtue of not satisfying (P4). As with (P1)/(P2), a divergent assignment works fine. Similar reasoning applies to (N3)/(N4). In both cases, however, there is nothing that favors one divergent semantic assignment over the other, yielding indeterminacy. And in both cases we cannot eliminate this indeterminacy via a demand for matching semantic values based on an appeal to symmetry.

Finally, the pathology of validity also resists reduction to just inconsistency via an appeal to symmetry, as demonstrated by the *asymmetric validity open pair*:[26]

> (V) ________1 = 1________.

[26] Arguments showing that (V) and (VI) have the same features as (III) and (IV) but cannot be understood as "saying the same thing" of each other can be found in Woodbridge & Armour-Garb, 2008, pp. 70-71.

∴ (VI) is invalid or (V) is
invalid

(VI) $\underline{\qquad\qquad 1 = 1 \qquad\qquad}$.
∴ (V) is invalid or ((VI) is
invalid and (V) is invalid)

The naysayer cases presented in this section strengthen the connection between semantic pathology's two symptoms; these pairs present single cases that manifest either pathological inconsistency or pathological indeterminacy. This unification of these different malfunctions provides further reason for taking them to be different symptoms of a single underlying problem. It would follow that a genuine solution to any particular case (e.g., the liar paradox) must ramify about the whole family. Moreover, the naysayer cases, in particular the asymmetric variants, systematically manifest a further symptom of semantic pathology, what we called *higher-order indeterminacy* while discussing the strengthened truthteller, (K+). With the naysayers, this further symptom arises independently of any appeal to truth-value gaps. Here it follows from the equi-possibility of these cases being inconsistent (by the members of each pair having the same semantic values) or *first-order* indeterminate (by the members of each pair being open to either of the possible divergent semantic-value assignments, with nothing to make them have one rather than the other), and there being nothing to decide between these two statuses. The significance of the naysayer cases thus goes beyond unifying semantic pathology's two basic symptoms and underwriting a demand for unified treatment of them. There is also a further type of resistance to semantic characterization that any adequate solution to semantic pathology must address. The following table collects all of these dual-symptom cases together for added impact.

Dual-Symptom Cases (Naysayers)

The Open Pair:

- (6) (7) is false
  (7) (6) is false

The Strengthened Open Pair:

- (8) (9) is not true
  (9) (8) is not true

The Curry Open Pair:

- (C1) (C2) is true $\rightarrow \perp$
  (C2) (C1) is true $\rightarrow \perp$

The Reference Open Pair:

- (N1) The thing(s) not referred to by the term labeled '(N2)'

  (N2) The thing(s) not referred to by the term labeled '(N1)'

The Satisfaction Open Pair:

- (P1) …does not satisfy the predicate labeled '(P2)'

  (P2) …does not satisfy the predicate labeled '(P1)'

The Validity Open Pair:

- (III) $\underline{\quad 1 = 1 \quad}$
  ∴ (IV) is invalid

  (IV) $\underline{\quad 1 = 1 \quad}$
  ∴ (III) is invalid

The Asymmetric Open Pair:

- (10) (11) is false
  (11) (11) is false $\rightarrow$ (10) is false

The Strengthened Asymmetric Open Pair:

- (12) (13) is not true
  (13) (13) is not true $\rightarrow$ (12) is not true

The Asymmetric Curry Open Pair:

- (C3) (C4) is true $\rightarrow \perp$
  (C4) [(C4) is true $\rightarrow \perp$] $\rightarrow$ [(C3) is true $\rightarrow \perp$]

The Asymmetric Reference Open Pair:

- (N3) The thing(s) not referred to by the term labeled '(N4)'

  (N4) The thing(s) either referred to by both (N3) and (N4) or not referred to by (N3)

The Asymmetric Satisfaction Open Pair:

- (P3) …does not satisfy the predicate labeled '(P4)'

  (P4) …satisfies both (P3) and (P4) or does not satisfy (P3)

The Asymmetric Validity Open Pair:

- (V) $\underline{\quad 1 = 1 \quad}$
  ∴ (VI) is invalid or (V) is invalid

  (VI) $\underline{\quad 1 = 1 \quad}$
  ∴ (V) is invalid or ((VI) is invalid and (V) is invalid)

5. *Responses to Semantic Pathology*

While we take the considerations and cases presented above to indicate that semantic pathology bifurcates into dual symptoms of inconsistency and indeterminacy, most programs for treating semantic pathology have taken inconsistency to be their main concern. The main division in approaches to pathological inconsistency is between *consistentists*, on the one hand, and *inconsistentists*, on the other. Consistentists call for a diagnosis of semantic pathology, together with a treatment that is meant to ensure that impending inconsistency will not arise. Inconsistentists accept the inconsistency *prima facie* inherent in the notion of truth and deal with it by revising, or at least restricting, its logical *consequences*. As consistentist responses have been the most prominent, we shall begin by discussing the two most influential of this kind of approach, Alfred Tarski's theory of truth and Saul Kripke's 'outline' for a theory of truth. We shall then turn to the most developed inconsistentist approach, Graham Priest's dialetheism.

Roughly speaking, for the semantic pathology that the liar paradox presents, consistentists fall under one or the other of the following general diagnostic camps: those who see the problem as arising from a defect in our language itself (herein, 'Camp I'), and those who see the problem as arising from a defect in our theorizing *about* our language (herein, 'Camp II'). Members of Camp I contend that natural languages are inconsistent, but they do not thereby embrace its apparent inconsistency. Rather, they explain the generation of paradox and attempt to regiment or revise either (i) certain features of the language—say, by providing an account of how the truth-*predicate* functions in the language—or (ii) certain semantic notions that apply to expressions of that language—say, by providing an account of the property of truth—in order to eliminate inconsistency and the paradox that it produces.

Members of Camp II do not base their theorizing on a problem with either a language or its semantic properties. Instead, they identify some premise in our reasoning with the semantic notions as giving rise to inconsistency (and thus paradox) and go on to revise our theorizing so as to avoid that premise. The central constraint on this approach is to avoid giving up certain features of our language that we had antecedently accepted.

In contrast with both consistentist camps, *dialetheists*, our representative inconsistentists, see the liar and related semantic paradoxes as yielding true contradictions. While they thus accept at least some contradictions,[27] they do not go on to deny that the paradoxes make evident some extant pathology. The difference in their view is that they take the pathology to be *logical*, as opposed to *semantic*.[28] Such theorists aim to 'contain' contradictions, by endorsing a logic that does not have an explosive consequence relation[29] inherent in classical logic. To this end, they propose a paraconsistent logic—one that denies *ex contradictione quodlibet* (herein, *ECQ*)—viz., from a contradiction, anything whatsoever follows—and allows for the logical possibility of true contradictions.[30]

In this section, we briefly discuss both camps of consistents, along with their respective treatments, ending with a brief discussion of dialetheism.[31]

5.1 *Consistentist Camp #1: Tarskian Consistentism and the Inconsistency of Natural Language*

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[27] No dialetheist maintains that all extant contradictions are true, just as no one—consistentist or not—would maintain that all sentence are true. For further discussion, see Priest, 1998.
[28] For more on logical and semantic pathology, see Armour-Garb & Woodbridge, 2006.
[29] A consequence relation, $\models$, is *explosive* if, and only if, for all formulae, P and Q, $\{P, \sim P\} \models Q$.
[30] Non-dialetheists can accept a paraconsistent logic, too. So-called 'relevance logicians' deny the validity of ECQ. They do not go on to affirm contradictions. Instead, they opt for a logic that takes a necessary condition for the consequence relation (and, thus, for the conditional) to involve *relevance*—either between antecedent and consequent or premises and conclusion. For more on dialetheism, see Armour-Garb, 2005 and Priest, 1987 and 2005. For more on paraconsistent logic, see Priest, 2002. For relevance logic, see Read, 1988.
[31] Briefly, our worry is that a dialetheist will have difficulty treating the cases of (what we call) 'higher-order indeterminacy'—viz., the *naysayers*. For more on this, see Armour-Garb & Woodbridge, 2006.

Alfred Tarski (cf. 1944/1983) seems to have held that natural languages are inconsistent, but maintained that we can eliminate the impending inconsistency (from *formalized* languages), by providing an elucidation, or a regimentation, of our language—one that captures the concept of truth, by providing a 'workable' account of the truth-predicate.[32]

He (1983, p.164) maintained that a central feature of a natural language is its 'universality' (herein, 'semantic universality'):

> A characteristic feature of colloquial language ...is its universality. It would not be in harmony with the spirit of this language if in some other language a word occurred which could not be translated into it; it could be claimed that 'if we can speak meaningfully about anything at all, we can speak about it in colloquial language.

He (Ibid., pp. 164-5) continues

> The common language is universal and is intended to be so. It is supposed to provide adequate facilities for expressing everything that can be expressed at all, in any language whatsoever; it is continually expanding to satisfy this requirement.

In order to avoid needless complications (e.g., entanglements regarding the expressibility, or inexpressibility, of properties), we can put forward a weaker notion of semantic universality—what we will call 'expressive completeness'—according to which, for a language, *L*, *L* is *expressively complete* if, and only if, it is able to express any feature that is possessed by an expression of *L*. So, for example, if English is expressively complete in this sense (and, thus, is weakly semantically universal), it will have the capacity to comment on the *statuses* of its own expressions, as well as on sentences that contain those expressions. In particular, *L* will be

[32] By *workable* we mean: a theory or proposal of the truth-predicate that, while sufficient to account for the function of the truth-predicate, will not necessarily capture the semantics of a language (e.g., English) of which the truth-predicate is a part.

capable of expressing, for any mentioned sentence of *L*, that it has the *semantic status* that it actually has.[33]

Tarski showed that, in light of the liar paradox, a language cannot consistently contain its own truth-predicate. As such, he (1983, pp. 164-5) took the liar paradox to provide "a proof that every language which is universal [in the above sense], and for which the normal laws of logic hold, must be inconsistent."[34] He did this by proving his *undefinability theorem* ("Tarski's theorem") according to which there can be no predicate of *L* true of all of the true sentences of *L*. Thus, there can be no predicate of *L* that represents truth-in-*L*.[35]

Tarski's treatment of the inconsistency of a natural language—his solution to the liar paradox—was to provide a highly artificial model of a language that was aimed at avoiding the paradoxes, while trying to retain as many of the features of the language as is possible.[36] In order to treat what he saw as inevitable semantic pathology, Tarski employed an object language/metalanguage distinction, where the latter provides resources adequate to discuss the former. To this end, the Tarskian theory of truth employs a *hierarchy of languages*, with the caveat that no level of the hierarchy contains a truth-predicate applicable to the sentences at that (or any higher) level (Cf. Tarski, 1944). So, the language at the first level of the hierarchy would not contain the truth (or the falsity) predicate; it would be a truth- (and falsity-) free language, containing all and only sentences of the *object language*—metaphorically, a language that is

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[33] This is not to say that *L* can *determine* which semantic status a given expression has; it is, rather, to say that if it is determined that a given expression of *L* has a particular status, then *L* will be capable of expressing that it does.

[34] The 'normal' laws of logic are the laws that codify 'classical logic'. As discussed below, one need not assume classical logic, in order to yield impending inconsistency (cf., the discussion of Kripke, below).

[35] As Tarski (1944) took his undefinability theorem to establish that a natural language like English is inconsistent, we might say that he aimed to show that our truth-predicate is in some way defective. As we might say, it is *expressively defective*, in that it is unable to express the property for which the predicate was introduced.

[36] Quine, 1960, p. 260 opts for, and introduces, this notion of a *regimentation*:

> We have, to begin with, an expression or form of expression that is somehow troublesome … But it also serves certain purposes that are not to be abandoned. Then we find a way of accomplishing those same purposes through other channels, using other and less troublesome forms of expression. The old perplexities are resolved.

used to talk about the world—as opposed to those of a metalanguage, which is used to talk about (e.g.) the language that is used to talk about the world. The second level of the hierarchy would contain a truth-predicate that applies to the true sentences at the first level; but—and this is crucial—that predicate, introduced in the second level of the hierarchy, does not apply to any sentence that is used in that level of the hierarchy. In general, an *n*-level language can contain the truth-predicate applicable to all of the true sentences of an $n - 1$ level language, though none at the n^{th} level. This restriction ensures that the impending theory of truth would be paradox-free. The end result was that, while Tarski maintained that truth is not definable in any language that contains its own truth-predicate, truth can nevertheless be explicated, given an infinite hierarchy of languages.

While Tarski's approach might be applicable to formalized languages, the idea of importing an infinite hierarchy and an infinite number of different truth-predicates into a natural language has struck many as unworkable.[37] Moreover, as Kripke pointed out, it is sometimes impossible to determine (or index) the relative levels of language pertaining to inter-related uses of the truth-predicate (e.g., cases like the open pair (6)/(7) above).[38] A further concern is that Tarski's approach appears susceptible to a revenge problem. Given Tarski's undefinability theorem, it seems that we cannot consistently express everything that there is to express. Indeed, even if we were to avail ourselves of a hierarchy of languages, one might still worry that paradox is unavoidable. For example, it does not seem that the appeal to a hierarchy of languages will enable us to avoid a liar sentence that says, of itself, that it is not true at any level of the hierarchy, as in

(L↕) (L↕) is not true at any level of the hierarchy.

[37] Van Fraassen, 1966; Martin, 1967; Kripke, 1975.
[38] Kripke, 1975, pp. 694-697.

A final concern we will mention is that one who views our language as unacceptably inconsistent and endorses a Tarskian (or a 'Tarski-like') response to the semantic paradoxes must also maintain that there are certain *prima facie* features of our language that we must now reject. For Tarski, it was semantic closure,[39] which arose from his claim about semantic universality.[40] Thus, what we learn from Tarski is that a language cannot consistently both contain its own truth-predicate and enjoy the expressive power that Tarski took a natural language to possess.

On its own, this conclusion is not obviously objectionable, but it appears to be worrisome, when we recognize that it appears to commit a theorist to an *error theory*, according to which certain principles and statements that we take to be true of, and about, a language like English end up as being uniformly untrue. Must we really give up semantic closure? Why not think, instead, that we have simply failed to land on an adequate theory of truth?

We take up this last question, when concluding this section. For now, we briefly introduce Kripke's theory, to document one answer to the penultimate question in the preceding paragraph.

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[39] Semantic closure is a technical term defined over formal languages. For present purposes, call a language, L, *semantically closed* if

(i) All expressions of L possess a semantic status;
and
(ii) Each of these semantic statuses can be expressed within L.

If we assume (we) and (ii), we will conclude that all well-formed sentences of L have *stated semantic statuses*—i.e., that each well-formed sentence of L has a designation and that L provides the means for expressing that such a well-formed sentence is evaluated as having that designation. For more on the role of semantic closure, in a discussion of consistent solutions to the semantic paradoxes, see Armour-Garb, 2005.

[40] For a language to be semantically universal, it must be able to explain the proper use of each of the expressions of that language. It is important to distinguishing 'semantic universality' from what is sometimes called 'general semantic universality', which holds that all *concepts* are expressible in a language. One might reject general semantic universality, while accepting semantic universality, since the former regards a thesis about *effability*, whereas the latter does not. Interesting though questions of effability are, we shall not have anything to say about it here, and, thus, will restrict attention to semantic universality, leaving issues that regard general semantic universality for another paper.

5.2 *Consistentist Camp #2: Kripkean Consistentism and the Inconsistency of Natural Language*

Saul Kripke (1975) has provided a theory of truth, which, if successful, would provide a means for defining a truth-predicate in a language for that language itself.[41] A central difference between Kripke's approach and Tarski's is that the latter depended on bivalence, assuming, that is, that every (meaningful) sentence is either true or false. Kripke goes beyond Tarski, by providing a formal theory of truth—an "outline" of such a theory, at any rate—and a logic that, in a sense, tolerates truth-value gaps. To this end, he assumes Kleene's (1952) three-valued logic—one that allows for three "statuses": true, false, and undetermined—sometimes called 'strong Kleene logic', K_3.[42] Under K_3, a sentence that is not judged to be true need not be judged to be false, for it may have an "undetermined", or an "unsettled", status.

Kripke's theory of truth showed that we could consistently maintain that a simple liar sentence, e.g.,

(L) Sentence (L) is false,

is undetermined, within a language that *in a sense* contains its own truth-predicate. What Kripke showed was that the relevant undetermined expressions were *ungrounded*, where, informally, an ungrounded sentence is one that is never assigned a determinate truth-value.

In order to explain the notion of ungroundedness, while also providing a Kripkean definition of 'semantic pathology', we briefly sketch Kripke's theory of truth.[43]

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[41] Kripke was not alone in attempting this; see, for example, Martin and Woodruff, 1975.

[42] K_3 is said to be strong in contrast with *weak* Kleene logic. On weak Kleene, for any truth function, if an *input* is a gap, then the output is a gap, too. By contrast, in K_3, if any of the inputs are classical (i.e., is either true or false) then the output is just as it was in the standard classical case. So, for example, on K_3 if P is false then P & Q will be false, no matter what value Q has. On weak Kleene, if either P or Q is gappy, then their conjunction will be gappy. Intuively, there will not be enough information to compute a truth-value—such a conjunction could then be said to be *undetermined*, 'unsettled', or the like.

[43] Kripke's theory of truth employs a very elegant mathematical framework. For purposes of space, we shall have to go forego details. The reader is encouraged to read Fitting, 1986 for more on the formal features of Kripke's theory.

Central to Kripke's theory of truth is the intuition that we are entitled to ascribe truth to a sentence when we are entitled to assert the sentence itself. Kripke (1975, p. 701) explains the intuition as follows.

> Suppose we are explaining the word 'true' to someone who does not yet understand it. We may say that we are entitled to assert (or deny) of any sentence that it is true precisely under the circumstances when we can assert (or deny) the sentence itself. Our interlocutor then can understand what it means, say, to attribute truth to [(14)] ('snow is white') but he will be puzzled about attributions of truth to sentences containing the word 'true' itself …

Nevertheless, with more thought the notion of truth as applied even to various sentences themselves containing the word 'true' can gradually become clear. Consider the sentence,

> (15) Some sentences printed in the *New York Daily*, October 7, 1971, is true,

which is a typical example of a sentence involving the concept of truth itself. If (15) is unclear, the same is true of

> (16) (15) is true.

However, our subject, if she is willing to assert that snow is white, will, according to the rules, be willing to assert that (14) is true. On the assumption that (14) is one of the sentences printed in the *New York Daily*, October 7, 1971, and our subject know this, then she will deduce (15) by existential generalization on her truth-attribution to (14). Once she is willing to assert (15), she will also be willing to assert (16). In this manner, the subject will eventually be able to attribute truth to more and more statements involving the notion of truth itself.

But there is no reason to suppose that all statements involving 'true' will become decided in this way, even if most will. Kripke does not maintain that *all* 'true'-involving sentences will be semantically evaluated; rather, he allows only that *some* will be semantically evaluated (or, as we prefer, *semantically characterized*). The ones that will be are the *grounded* sentences; those that will not are the *ungrounded* sentences. Ungrounded sentences are those sentences that, through the afore noted process, are never assigned a truth-value. This gappiness is then transferred to the truth-predicate itself, which Kripke takes to be partially defined, in virtue of the fact that some sentences, being neither true nor false, will neither be those to which the truth-predicate truly applies nor those to which the truth-predicate falsely applies. In jargon, there will be some sentences that are neither in the extension, nor the anti-extension of 'true'.

Intuitively, an ungrounded sentence is one that a language is unable properly to characterize; a grounded sentence is one that a language can properly characterize. In order to get a formal account of groundedness, Kripke employs a hierarchy of first-order languages, each containing both the syntax and the semantics that are needed, in order to contain 'true'-involving sentences of the sort that were noted in Kripke's quote. Although Kripke employs a hierarchy, in his formal construction, we should see the hierarchy as *in effect* internalized within a single language.

Following Kripke, let us stipulate a language-fragment, L_0, containing a class of well-formed sentences, none of which include the truth- or falsity-predicate.[44] As we might say, L_0 is a *truth-free language-fragment*. We can say that this fragment of our language is at the first level of the hierarchy, if we like, but that is liable to confuse, since Kripke is not relying on a hierarchy. Let us say, then that L_0 is a language at the first stage—that is, that L_0 is a *language at stage 0*.

[44] Kripke (1975) takes L_0 be a classical, first-order language, which includes a collection of primitive predicates—those that do not include the truth-predicate.

While some sentences of L_0 will be true (e.g., 'snow is white') and other sentences of L_0 will be false (e.g., 'snow is magenta'), no such sentences can be characterized in L_0 as true or false, for L_0 does not include any sentences that contain the truth-predicate. Suppose, then, that we expand L_0, by adding a truth-predicate, τ, and expand our language to another stage, to arrive at L_1. At L_1, the extension and anti-extension of τ is non-empty, for the extension will contain all of the true sentences of L_0 and the anti-extension will contain all of the sentences of L_0 that are not true.

As the languages extend, the extension and the anti-extension of τ expands. Moreover, if a sentence is determined to be true at one language of the "stage construction", it will be characterized as true (and will remain to be true) by each of the levels that are "above it" in the succession of stages—and the same applies for sentences that are determined not to be true of an L_i at the relevant *i*-stage.[45]

At some point, the process by which the extension and anti-extension of τ increases stops, which is to say that there is a point—a *minimal fixed point*— at which all of the sentences that are determined to be true at that stage are in the extension of τ at that stage; and all of the sentence the are determined to be false at that stage are in the anti-extension of τ at that stage—and every sentence that is true (false) is ascribed truth (falsity). A language in which the set of truths (and the set of sentences that are not true) of that language is identical to the extension (anti-extension) of the truth-predicate of that language is called *a fixed-point language*. What Kripke showed was that if we start with a language at a stage in which the extension and anti-extension of the truth-predicate is empty, there will be a *minimal fixed point* for our language.

[45] The relevant function that yields this result is *monotonic*.

At the minimal fixed point, our language contains its own truth-predicate, thereby appearing to satisfy (at least weak) semantic universality.

What about the liar, the truthteller, the open pair and the like? One important feature of Kripke's construction is that if a sentence is not determined to be true (or not) at the first stage, by applying Kripke's process, it will never be in the extension or the anti-extension of τ. Thus, a sentence that falls out of the extension or anti-extension of τ at L_1—viz., a *gappy* sentence—will not have a truth-value at the minimal fixed point. Kripke maintained that all such sentences are *ungrounded*, where this notion can now be formally defined: A sentence, *s*, is ungrounded if, and only if, it is not a member of the extension or the anti-extension of the truth-predicate at the minimal fixed point.

It is easy to see that, for Kripke, the simple liar,

(L) Sentence (L) is false

will never receive a truth value at any fixed point. Now, since (L) will neither be in the extension or the anti-extension of τ at L_1, it will not be said to be true or false at the minimal fixed point.[46] Indeed, (L) cannot have a truth-value at any fixed point, for it cannot be determined to be true (false) at any stage without its being in the extension and anti-extension of τ at a given fixed point. So, on Kripke's theory, (L) (and liar-like variants) *falls out* of the extension and the anti-extension of the truth-predicate. Accordingly, we can say that Kripke delivers a predicate for a language, which is within that language, that is *partially defined*—one that is true of the true sentences, false of the false ones, and silent on those that that are neither true nor false.

[46] We can say (if we like) that the sentence is gappy. But it is important to note that this is not to say that (L) has a "3rd value", that is, a third condition had by sentences that fall out of truth's extension and anti-extension.

Let us say that a sentence is ungrounded if it does not receive a truth-value at the minimal fixed point and paradoxical if it does not receive a truth-value at any fixed point. The simple liar is ungrounded and paradoxical. The truthteller, (K), is ungrounded but non-paradoxical. Recall that (K) *can* be true or false without contradiction, though neither aletheic predicate has more claim on it that the other. Following Kripke's construction, (K) will not receive a truth-value at the minimal fixed point; thus, it is ungrounded. But, unlike (L), (K) can have a truth-value at different, non-minimal, fixed points; so, it is not paradoxical.

To say that (K) can have a truth-value at a non-minimal point is to say that, without threat of contradiction, we can arbitrarily assign a truth-value to (K) at a level, given some stage of Kripke's process. If, say, we let (K) be true at a given level for a language then there will be a fixed point at which it is true in that language; if, in a different construction, we let (K) be false at a given level (and, still, for a language) then there will be a fixed point at which it is false in that language. Thus, (K) is ungrounded but non-paradoxical: It has no determinate truth-value for a language that contains its own truth-predicate, though, given Kripke's construction, it can have truth-values at non-minimal fixed points.

Let us return to the open pair. On Kripke's theory of truth, the sentences in the open pair,

(6) (7) is false
(7) (6) is false

are likewise ungrounded. It is a bit trickier to determine if they are paradoxical. First, (6) and (7) do not have the same truth-value at any fixed point, for the same reason that (L) does not have any truth at any fixed point. That said, while the pair can have divergent truth-values at different fixed points, they receive no such assignments at a minimal fixed point. Thus, they are ungrounded. But notice that Kripke's theory of truth does not obviously determine whether they are also paradoxical, for that depends on what we must say about the members of both the symmetrical and the asymmetrical pair. This is certainly not a problem for Kripke's theory, for, being ungrounded, both (6) and (7) (and their asymmetrical variants, (10) and (11)) will fall out of the extension or anti-extension of the truth-predicate. Such sentences are *at least* semantically undetermined, and, as we will say, are thus at least *semantically pathological*.[47]

As noted, Kripke's aim was to provide a theory of truth for a language that was (weakly) semantically universal. Thus far, it seems that he has succeeded. But, even recognizing the importance of semantic universality, there is another feature that we would like our theory to honor—that of *semantic completeness*. The problem with Kripke's theory is that he can retain a semblance of semantic universality, but only at the cost of semantic incompleteness, via a denial of semantic universality.

In order to see this, consider a *strengthened liar*,

(L+) Sentence (L+) is not true,

where, intuitively, a sentence is not true if it is either false—and, thus, is in the anti-extension of "true"—or, at least, is *not* in the extension of the truth-predicate. Clearly, (L+) will not receive a truth-value at a minimal fixed point; hence, it is ungrounded. But, unlike the truthteller, and given what it says, it seems that it is not the case that (L+) is true. The pressing question is whether a fixed-point language can construct what is, *in effect*, an untruth-predicate—an expression whose extension consists of all of the sentences that are not in the extension of the truth-predicate.

Without contradiction, (L+) cannot be in the extension or the anti-extension of τ at the minimal fixed point. If there could be such an untrue predicate, (L+) would be a member of its

[47] Kripke (1975) seemed to take the minimal fixed point to be central to his theory of truth; accordingly, the question as to whether the open pair sentences are paradoxical would not be of much interest. We briefly take up this issue, below.

extension, in which case it will wind up in the extension of τ.[48] Hence, if there is such a predicate, it seems that paradox is unavoidable, unless we are prepared to deny semantic universality. Moreover, given that we seem able to say that (L+) is not true (since it seems that we *know* that (L+) cannot be true), we risk losing semantic completeness.

With these points taken as read, the problem posed by (L+) is evident: An advocate of a Kripkean account avoids contradiction only if she accepts that the language is expressively incomplete, in which case the failure of (weak) semantic universality follows. If, however, Kripke (or, at least, a Kripkean) wishes to retain expressive completeness (and thus semantic universality), he will find that he can do this only by countenancing some sort of hierarchy in the language. Kripke (1975/1984, p.80) recognizes the problem, admitting that what the dilemma makes evident is that "the ghost of the Tarski hierarchy is still with us." [49] Thus, while Kripke's outline greatly improves on a Tarskian theory (and is, anyway, of tremendous importance to mathematics and logic), it is still the case that expressive completeness is sacrificed, in the service of maintaining consistency.

5.3 *Inconsistentist Approaches: Dialetheism and the Inconsistency of Natural Language*

We have indicated some serious concerns about the adequacy of the two main consistentist approaches to semantic pathology. Perhaps an entirely different kind of approach is what is needed, in particular, one that does not attempt to eliminate the inconsistency that our semantic notions exhibit. We mentioned above that Priest's dialetheism is the most developed inconsistentist approach, so it shall be our focus. Dialetheism is the view that some meaningful sentences (or, generally, truth bearers) are true whose negations are likewise true. Given the relation between falsity and truth of negation, it follows that such sentences are both true and

[48] So, is (L+) also paradoxical, in addition to being ungrounded? As is clear, below, the question of whether it is paradoxical is intimately linked up with questions regarding semantic completeness.

[49] Soames, 1998 has attempted to retain Kripke's theory of truth without need to ascend to a metalanguage.

false.[50] A *dialetheia*—from the Greek, a *two-way truth*—is a true contradiction, a truth bearer, A, such that both A and its negation, ~A, are true. Accordingly, dialetheism is the view that there are true contradictions.

An immediate concern is that according to ECQ, everything follows from a contradiction, so if any contradiction were true, that would appear to make every sentence true, yielding trivialism. Logical revision (away from classical logic, which includes ECQ) is required to block this result. What is needed is a logic that does not include ECQ, or, more colorfully, a logic that does not include "explosion". Logics for which explosion fails are *paraconsistent logics*.[51] Let $\models$ be a relation of logical consequence, either semantically or proof-theoretically defined. We can say that $\models$ is explosive if, and only if, for all formulae, Φ and Ψ, $\{\Phi, \sim\Phi\} \models \Psi$, and that a logic is paraconsistent if, and only if, its notion of logical consequence is not explosive.[52] Not all paraconsistent logics sanction contradictions (e.g., Relevance Logic does not). One paraconsistent logic that does allow for contradictions to have a designated logical value is *LP*, "the logic of paradox," which was formally introduced by Graham Priest (1979, 1984).

One of the strongest cases for dialetheism arises from the *prima facie* appearance that English is *semantically closed*. Kripke's outline notwithstanding, few philosophers would reject the claim that English is semantically closed, at least prior to confronting the liar paradox, (L).[53] Upon confronting the liar paradox, however, the traditional reaction has involved rejecting appearances. That is, it involves rejecting, to some degree or other, the claim that English can express its own semantics.

[50] This subsection loosely follows Armour-Garb, 2005.
[51] For more on paraconsistent logics, see Priest, 2002.
[52] For the details, see Priest, 1987; Priest, Beall & Armour-Garb, 2003; and Woods, 2003.
[53] We should note that both dialetheists and consistentists are aware of the fact that they owe an enormous debt to Kripke. In effect, they argue that semantic closure, universality, and the like are features that we cannot give up, even when confronted with ensuing inconsistency. For more on the debt, see Beall & Armour-Garb, 2005 Armour-Garb, 2005, and Priest, 2005.

The problem with this emerges as follows. It begins with rejecting the strong appearance of semantic closure, but then revenge problems (such as the move from (L) to (L+) in the face of a proposed "gaps" approach to (L)) indicate that still other appearances must be rejected. The typical cases involve sentences that, given a proposed (consistent) semantic theory, appear to be true—or appear to be such that the theory ought to deem them to be true. On pain of inconsistency, however, the relevant theory cannot respect appearances. Thus, we purchase our consistency by rejecting appearances that, it seems, we *ought* to accept.[54] So goes *the consistentist's bind*, which fuels the dialetheist's fire.

5.3.1. Dialetheism and the Open Pair

Dialetheism provides an account of the liar paradox and other cases of inconsistent semantic pathology, but it is not clear that it can deal with the cases of indeterminate semantic pathology, or the dual-symptom cases. The latter, open pair cases pose the biggest challenge for dialetheists, since the approaches to them that they have offered cannot account for all of the cases, and thus leave cases of semantic pathology unresloved.

Following most consistentists—from Buridan and Bradwardine to the present—Priest (2005) presents an account of the open pair that is based on an appeal to symmetry, maintaining that

> [t]he situation concerning [the sentences of the open pair] is, in all respects, symmetrical; it cannot, therefore, have an asymmetric upshot. Either both sentences are true, or both are false… Hence, it would seem, both sentences [of the open pair] are true and false [or true and not true].[55]

[54] For more on this construction of semantic closure, see Armour-Garb & Beall, 2001.
[55] For further discussion, see Armour-Garb & Woodbridge, 2006.

Following Roy Sorensen (1998),[56] Priest claims that the sentences of the open pair have truth-values; but, unlike Sorensen, he endorses symmetry, in which case we will find that the members of the open pair,

(8) (9) is not true
(9) (8) is not true,

are both true and not—a true contradiction. If we ignore extraneous worries about dialetheism (as we will) and grant the possibility of that view, it seems that we should conclude, at least p*rima facie*, the putative semantic pathology that appears to plague the aforementioned consistentists does not filter down to dialetheism, as it seems that the latter can treat these cases as inconsistent semantic discourse that help reveal, at best, an initial logical pathology.

In fact, however, it seems that the dialetheist is *not* in a good position to resolve, even *inconsistently*, all the variants of the open pair. To see why, suppose, following Priest, that we are to accept both symmetry and inconsistency. This seems to give the dialetheist an advantage in resolving any of (6)/(7) and (8)/(9), but what about the following, revenge-related pair,

(12) Sentence (13) is not true
(13) If sentence (13) is not true, then sentence (12) is not true?

To be sure, the dialetheist *can* ascribe them matching (and, thus, inconsistent) truth-values, and, if they do, can assimilate these cases with the liar. But he can also ascribe them divergent (and, thus, consistent) truth-values, with no independent reason to favor one of these approaches over the other. Moreover, with the asymmetric cases the dialetheist seems to face two levels of indeterminacy: (i) an indeterminacy regarding whether to make a consistent or an inconsistent truth-value attribution; and, if a consistent ascription is favored, (ii) an indeterminacy regarding which sentence of a given pair is true and which is false.

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[56] For a conclusive rejection of Sorensen's treatment of the open pair, see Woodbridge & Armour-Garb, 2005 and Armour-Garb & Woodbridge, 2006 and 2010.

As we have shown elsewhere,[57] there is no viable means by which Priest can resolve the pathology that the asymmetric variants of the open pair present. Thus, even if the liar paradox can be (inconsistently!) resolved, cases of semantic pathology remain, neither adequately diagnosed nor adequately treated by dialetheism. As such—and in light of the problems with consistent solutions to the semantic paradoxes—we contend that an alternative approach would be desirable—one that incorporates the positive insights found in consistent and dialetheic solutions to semantic pathology, without falling victim to any of the problems that plague them. Although we have such an approach, we shall have to leave it for another time.

[57] Armour-Garb & Woodbridge, 2006.

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